Objectives

1. To introduce the concept of the free-body diagram for a particle.
2. To show how to solve particle equilibrium problems using the equations of equilibrium.

Definitions

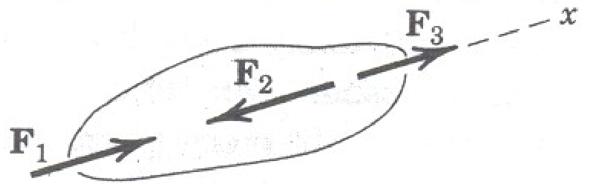
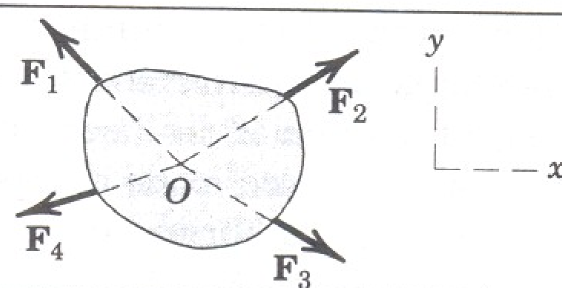
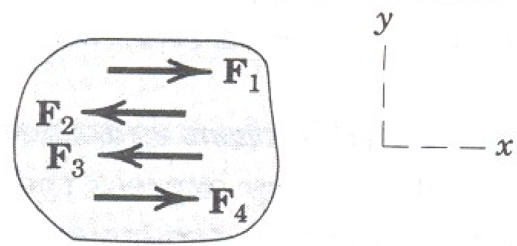
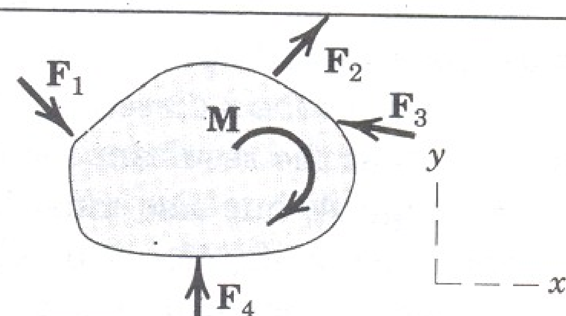
1. A particle is in equilibrium if it is at rest if originally at rest or has a constant velocity if originally in motion.
2. Static equilibrium denotes a body at rest.
3. Newton’s first law is that a body at rest is not subjected to any unbalanced forces.

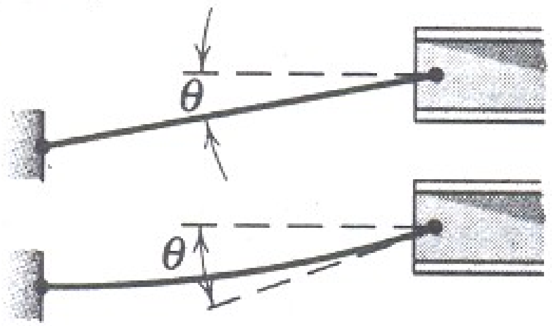
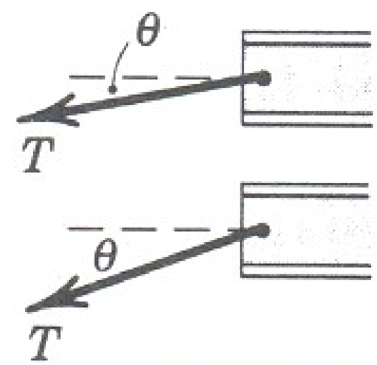
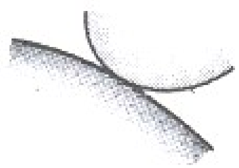
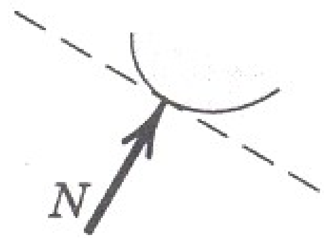
Static Equilibrium

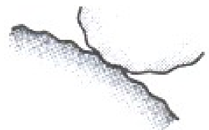
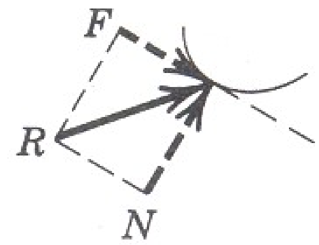
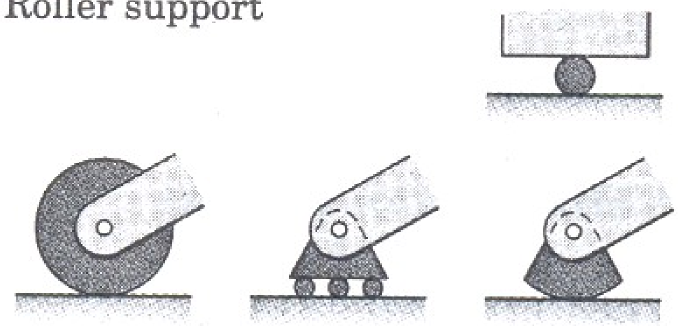
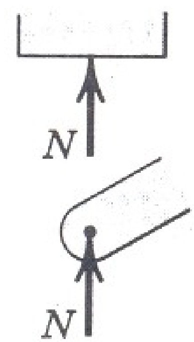
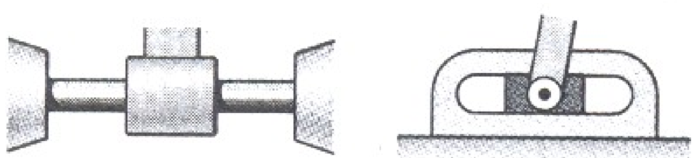
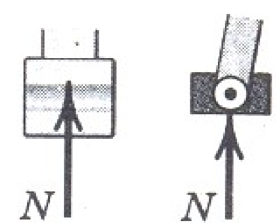
$$\boxed{\sum \vec{F} = 0}$$

Static Equilibrium

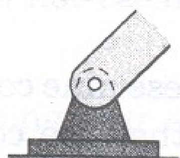
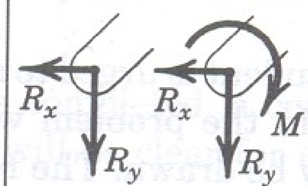
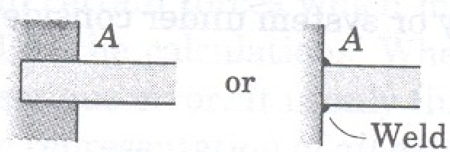
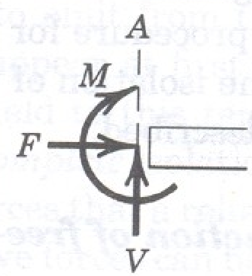
$\sum \vec{F}$ *is the vector sum of all forces acting on the particle.*

CATEGORIES OF EQUILIBRIUM IN TWO DIMENSIONS		
Force System	Free–Body Diagram	Independent Equations
1. Collinear		$\Sigma F_x = 0$
2. Concurrent at a point		$\Sigma F_x = 0$ $\Sigma F_y = 0$
3. Parallel		$\Sigma F_x = 0$ $\Sigma M_z = 0$
4. General		$\Sigma F_x = 0$ $\Sigma M_z = 0$ $\Sigma F_y = 0$

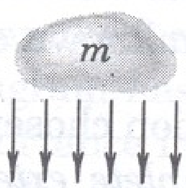
Type of Contact and Force Origin		Action on Body to be Isolated	
1. Flexible cable, belt, chain, or rope Weight of cable negligible Weight of cable not negligible			Force exerted by a flexible cable is always a tension away from the body in the direction of the cable.
2. Smooth surfaces			Contact force is compressive and is normal to the surface.

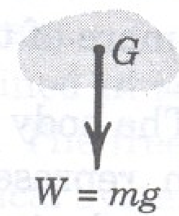
3. Rough surfaces			Rough surfaces are capable of supporting a tangential component F (frictional force) as well as a normal component N of the resultant contact force R.
4. Roller support			Roller, rocker, or ball support transmits a compressive force normal to the supporting surface.
5. Freely sliding guide			Collar or slider free to move along smooth guides; can support force normal to guide only.

MODELING THE ACTION OF FORCES IN TWO-DIMENSIONAL ANALYSIS (*cont.*)

Type of Contact and Force Origin	Action on Body to be Isolated	
6. Pin connection 	 	A freely hinged pin connection is capable of supporting a force in any direction in the plane normal to the axis; usually shown as two components R_x and R_y. A pin not free to turn may also support a couple M.
7. Built-in or fixed support  	 	A built-in or fixed support is capable of supporting an axial force F, a transverse force V (shear force), and a couple M (bending moment) to prevent rotation.

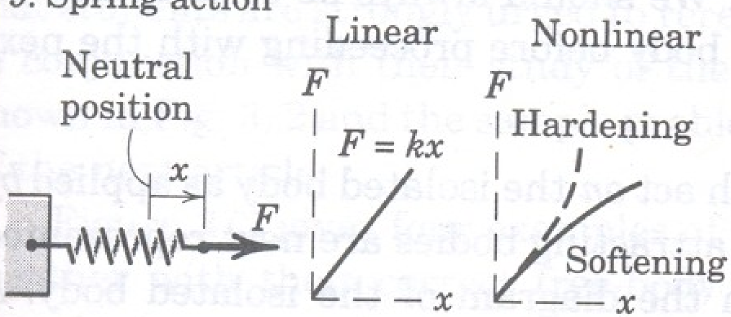
8. Gravitational attraction

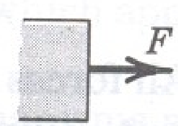




The resultant of gravitational attraction on all elements of a body of mass m is the weight $W = mg$ and acts toward the center of the earth through the center mass G.

9. Spring action

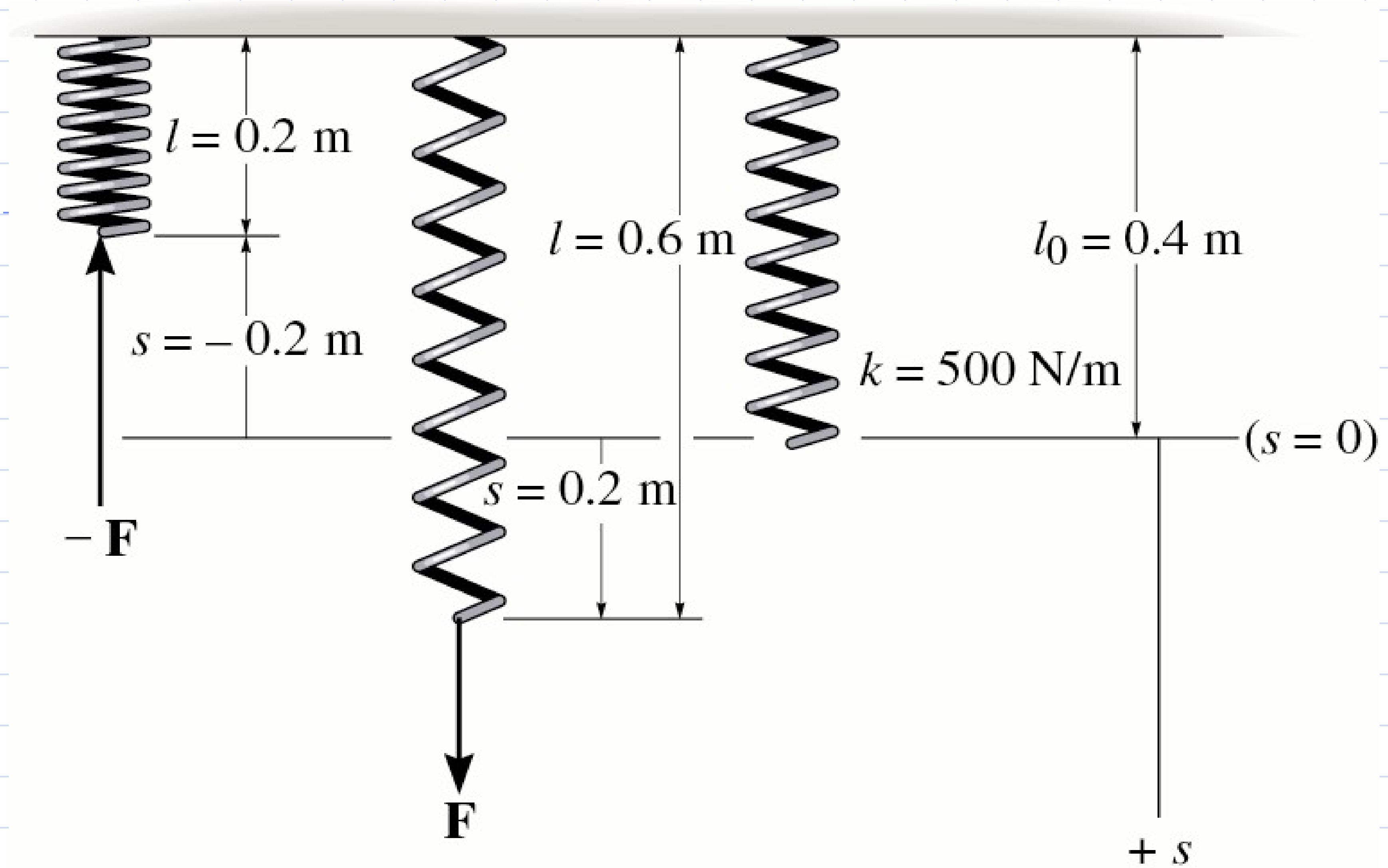




Spring force is tensile if spring is stretched and compressive if compressed. For a linearly elastic spring the stiffness k is the force required to deform the spring a unit distance.

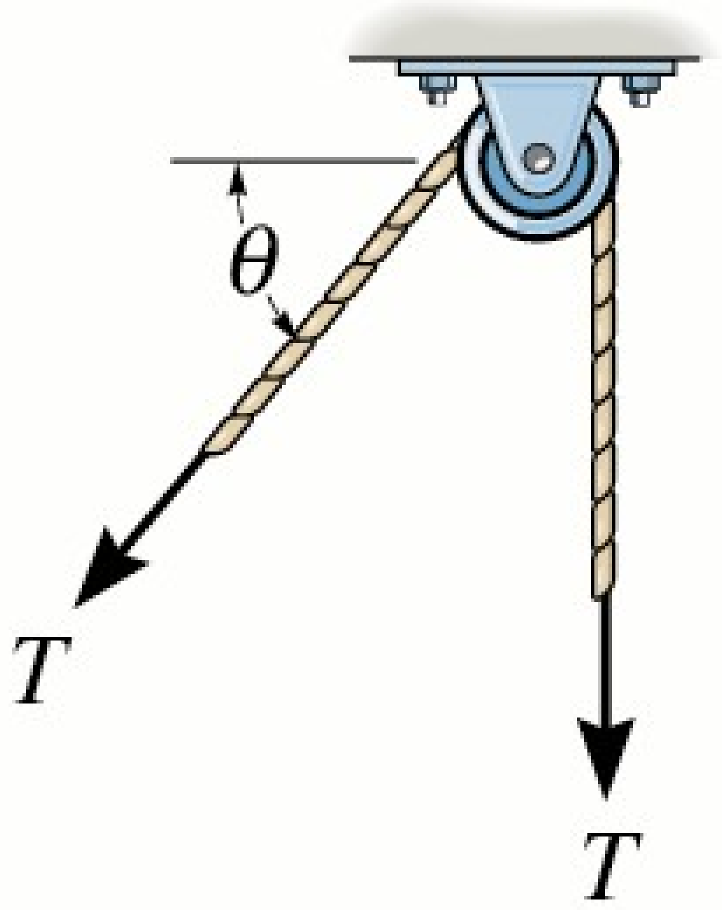
Springs

$$F = ks$$



$l = 0.2$ m
$s = -0.2$ m
$-\mathbf{F}$
$l = 0.6$ m
$s = 0.2$ m
$\mathbf{F}$
$l_0 = 0.4$ m
$k = 500$ N/m
$(s = 0)$
$+ s$

Cables and Pulleys



Cable is in tension

Cables and Pulleys

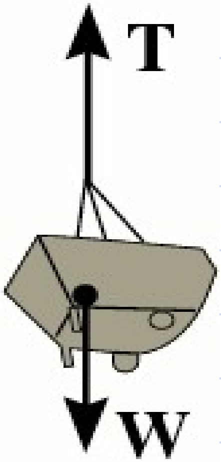
Cables are assumed to have negligible weight and they cannot stretch. They can only support tension or pulling ***(you can't push on a rope).*** **Pulleys are assumed to be frictionless. A continuous cable passing over a frictionless pulley must have tension force of a constant magnitude. The tension force is always directed in the direction of the cable.**

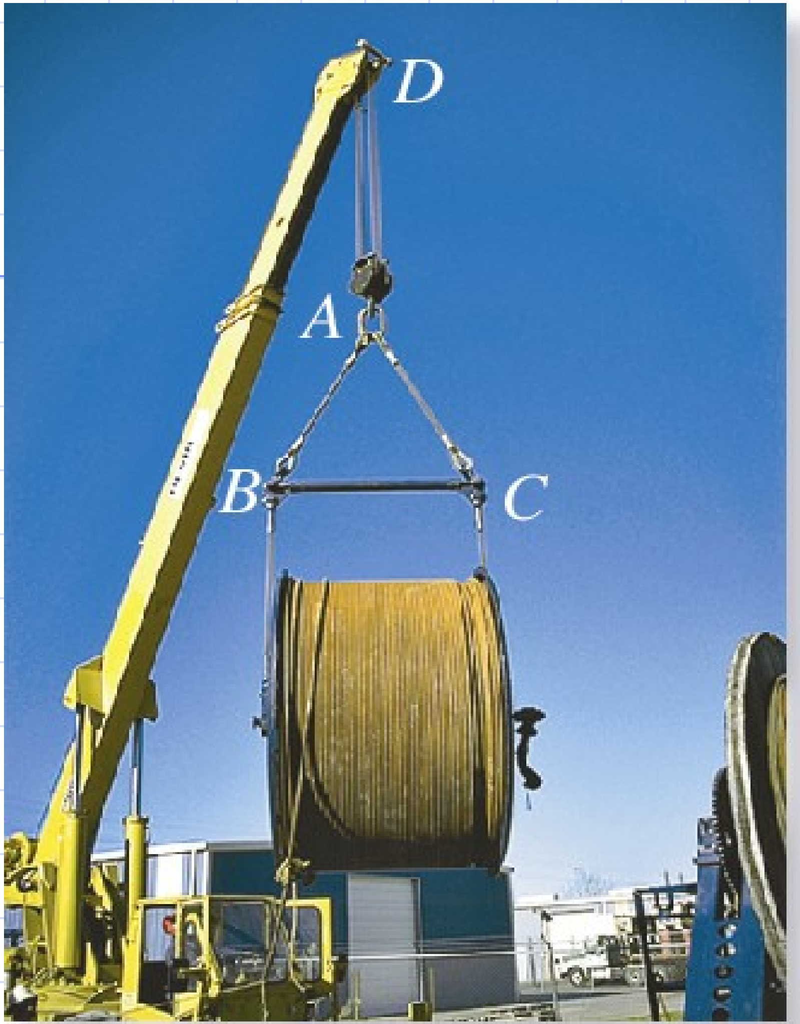
The Free-Body Diagram

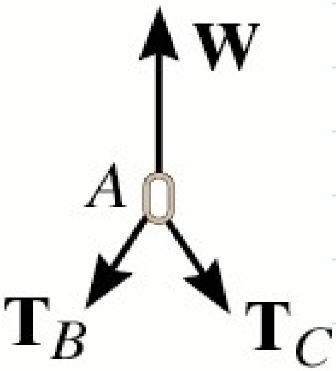
1. To apply equilibrium equations we must account for all known and unknown forces acting on the particle.
2. The best way to do this is to draw a free-body diagram of the particle.
3. The free-body diagram (FBD) of a body is a sketch of the body showing all forces that act on it. The term free implies that all supports have been removed and replaced by the forces (reactions) that they exert on the body.

Drawing Free-Body Diagrams

1. Draw Outlined Shape - Imagine the particle isolated or cut “free” from its surroundings
2. Show All Forces - Include “active forces” and “reactive forces”
3. Identify Each Force - Known forces labeled with proper magnitude and direction. Letters used for unknown quantities.


T
W

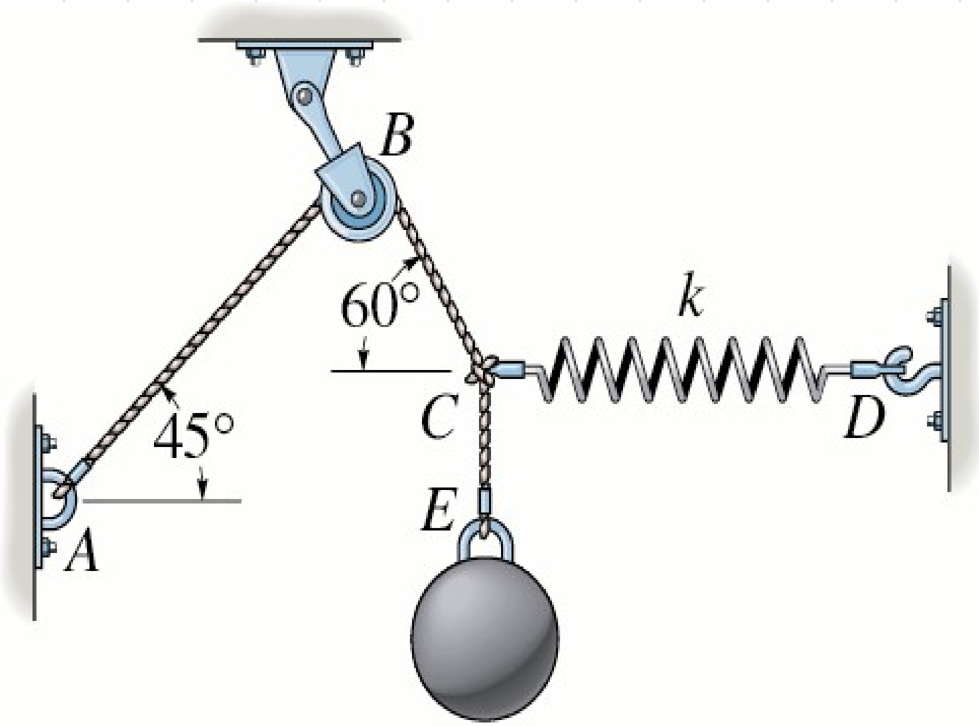

D
A
B
C


W
A
T_B
T_C

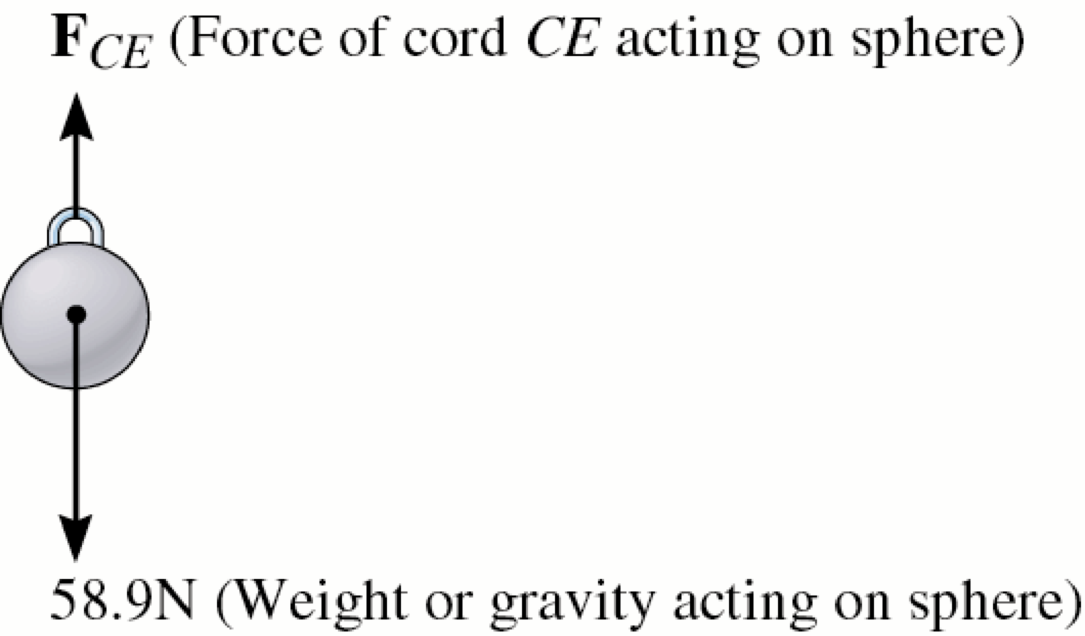
Force Types

1. Active Forces - tend to set the particle in motion.
2. Reactive Forces - result from constraints or supports and tend to prevent motion.

Example



The sphere has a mass of 6 kg and is supported as shown. Draw a free-body diagram of the ***sphere, cord CE***, and ***the knot at C***



Sphere

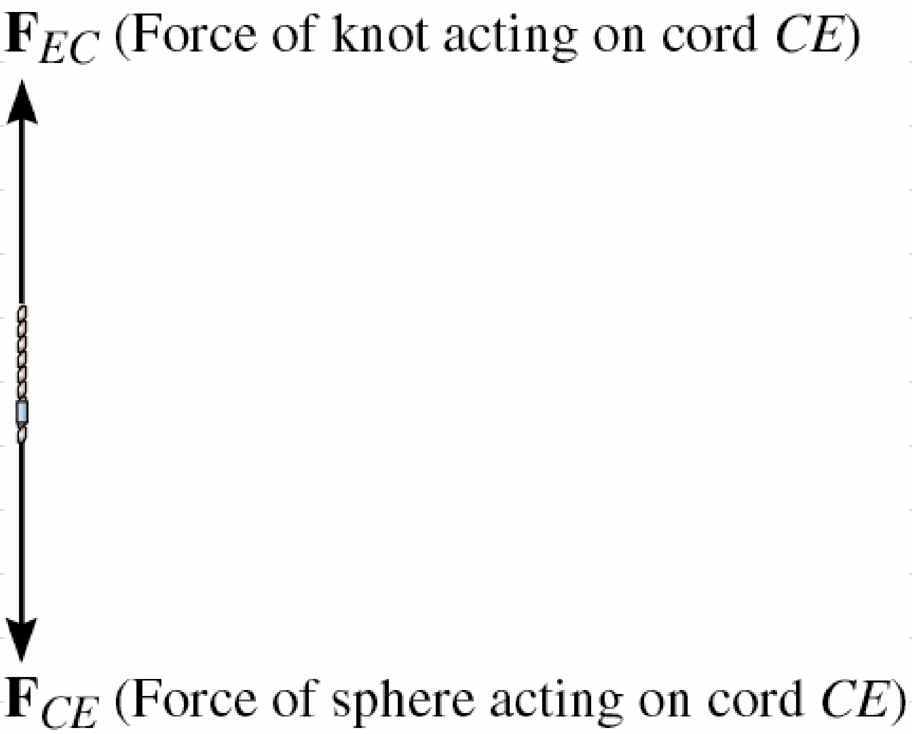
There are two forces acting on the sphere. These are its weight and the force of cord CE.

The weight is:

W = 6 kg (9.81 m/s^2)=58.9 N

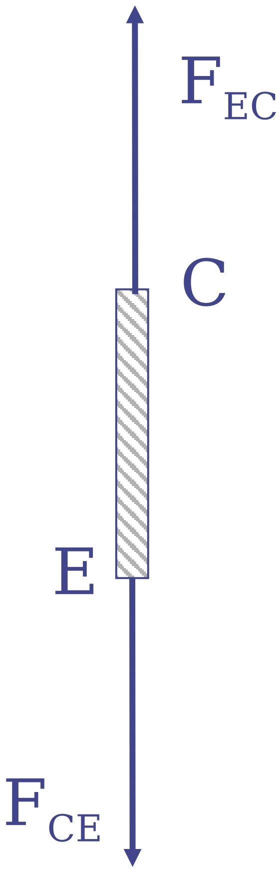
Sphere

Cord CE



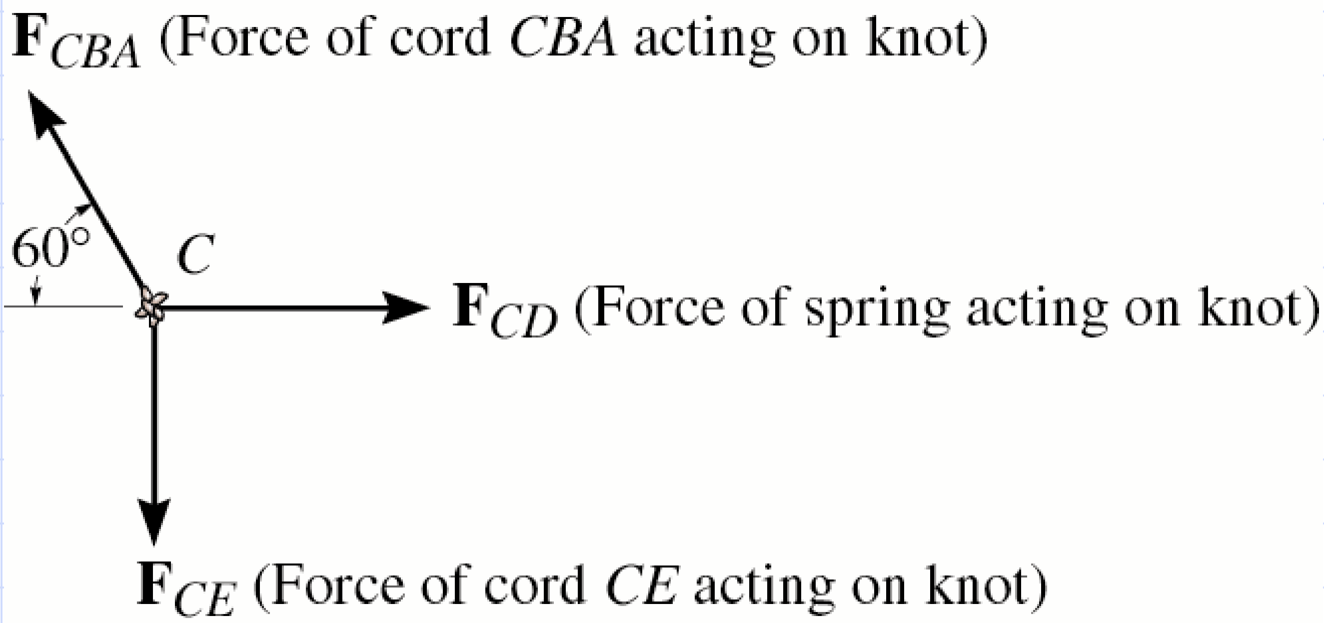
There are two forces acting on the cord. These are the force of the sphere, and the force of the knot. A cord is a tension only member. Newton's third law applies.

Cord CE



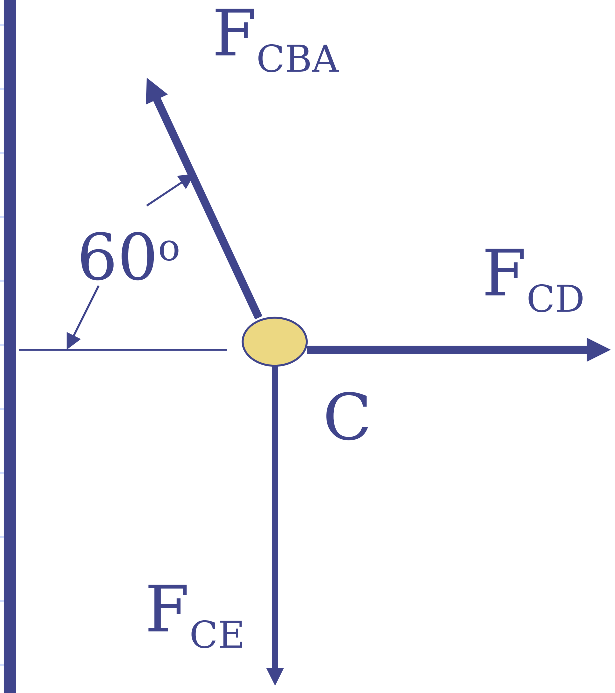
Free-Body Diagram

Knot at C



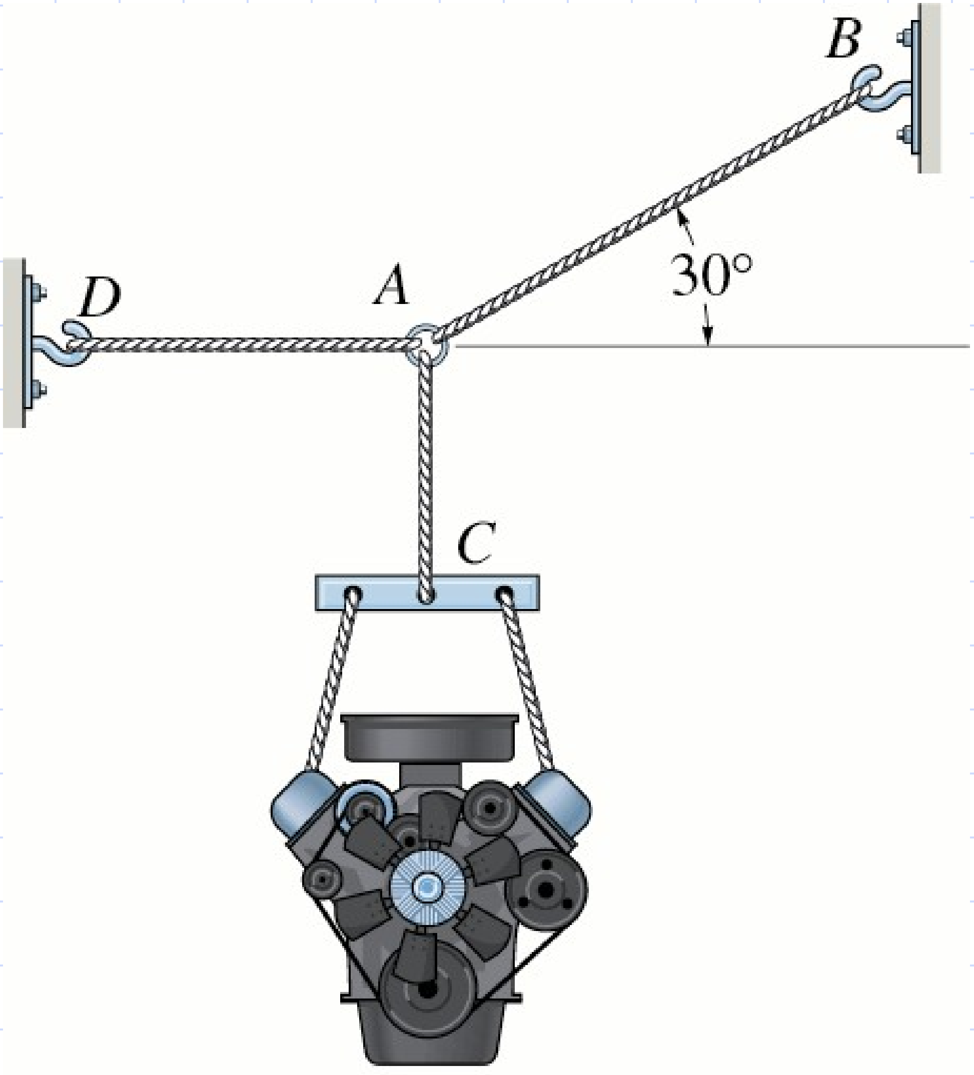
There are three forces acting on the knot at C. These are the force of the cord CBA, and the force of the cord CE, and the force of the spring CD.

Knot at C

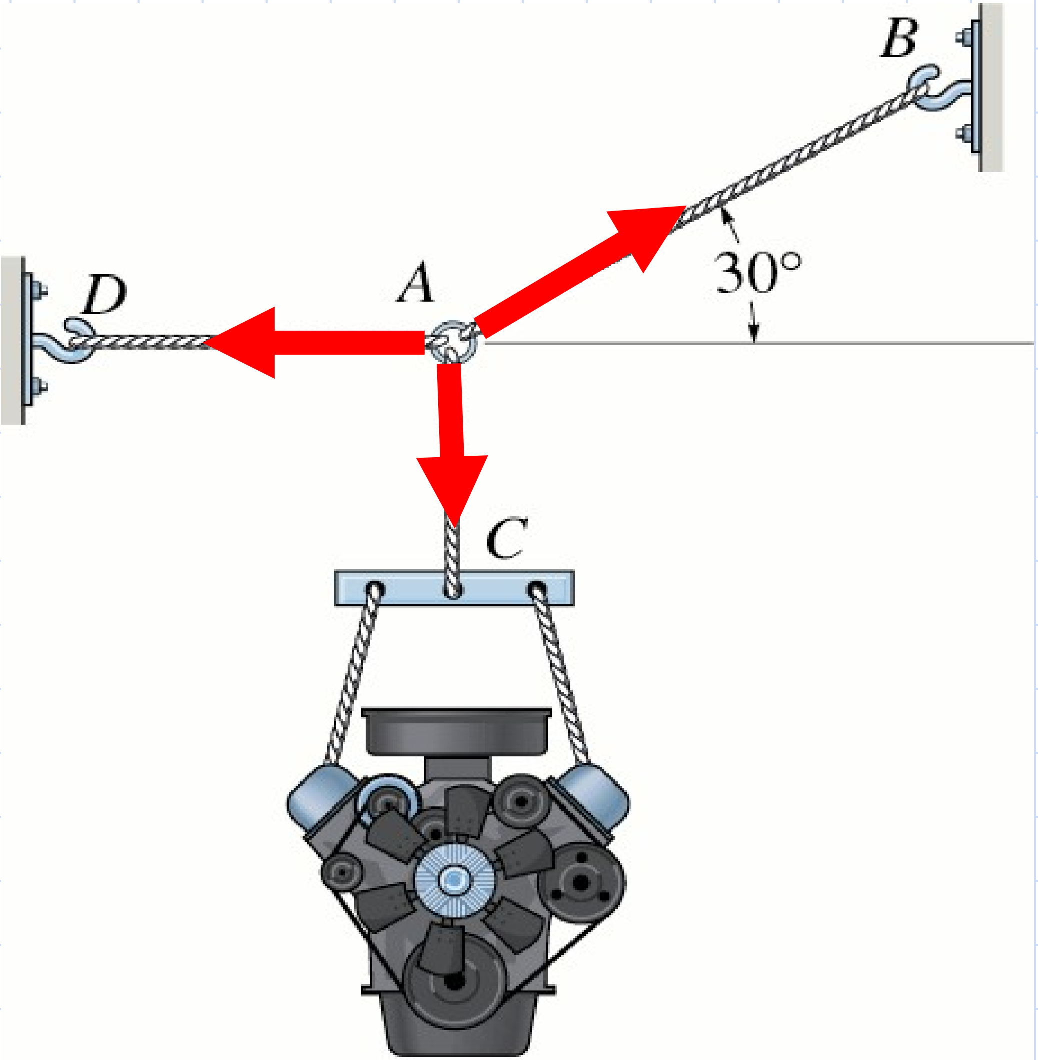


Free-Body Diagram

Example

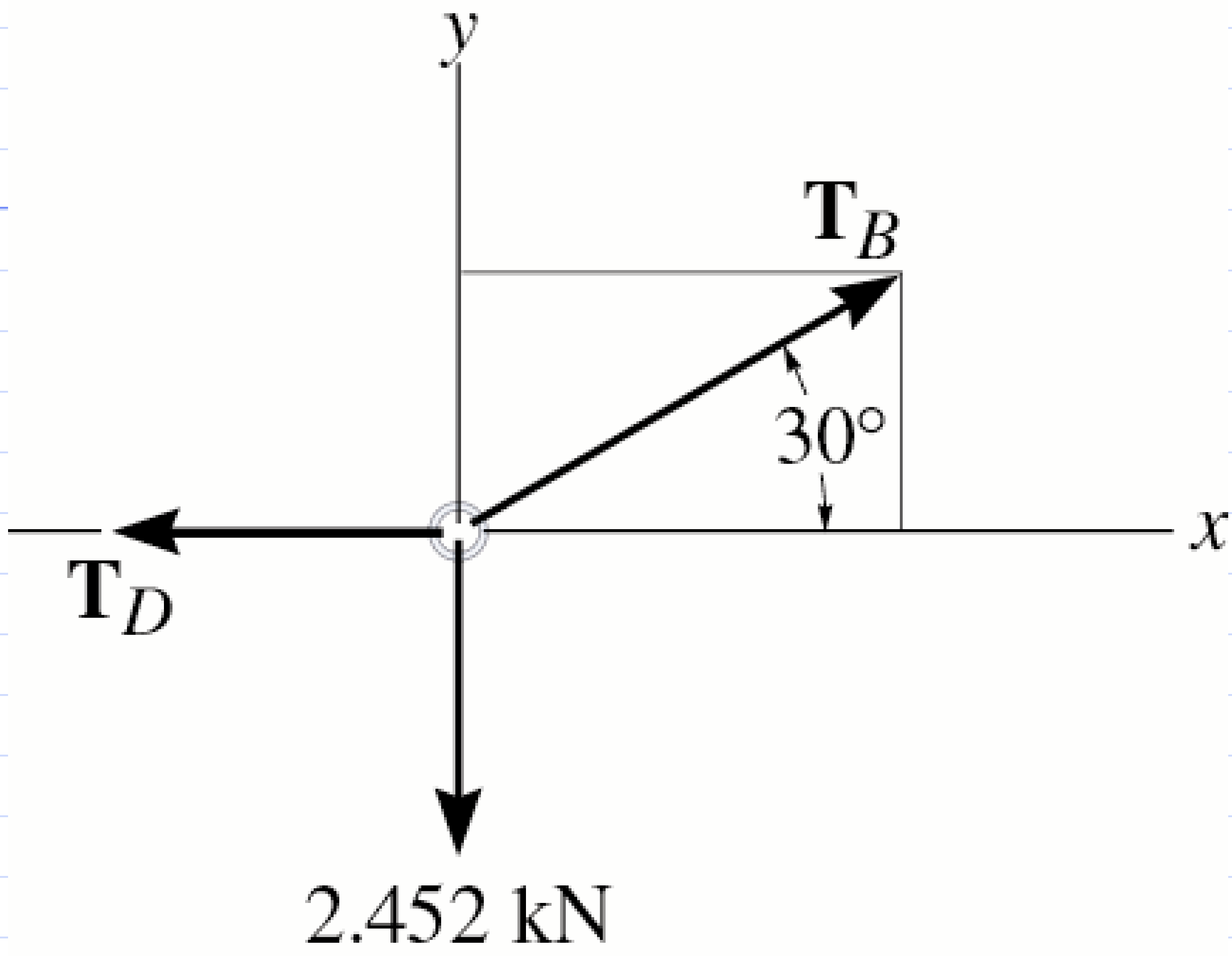


Example

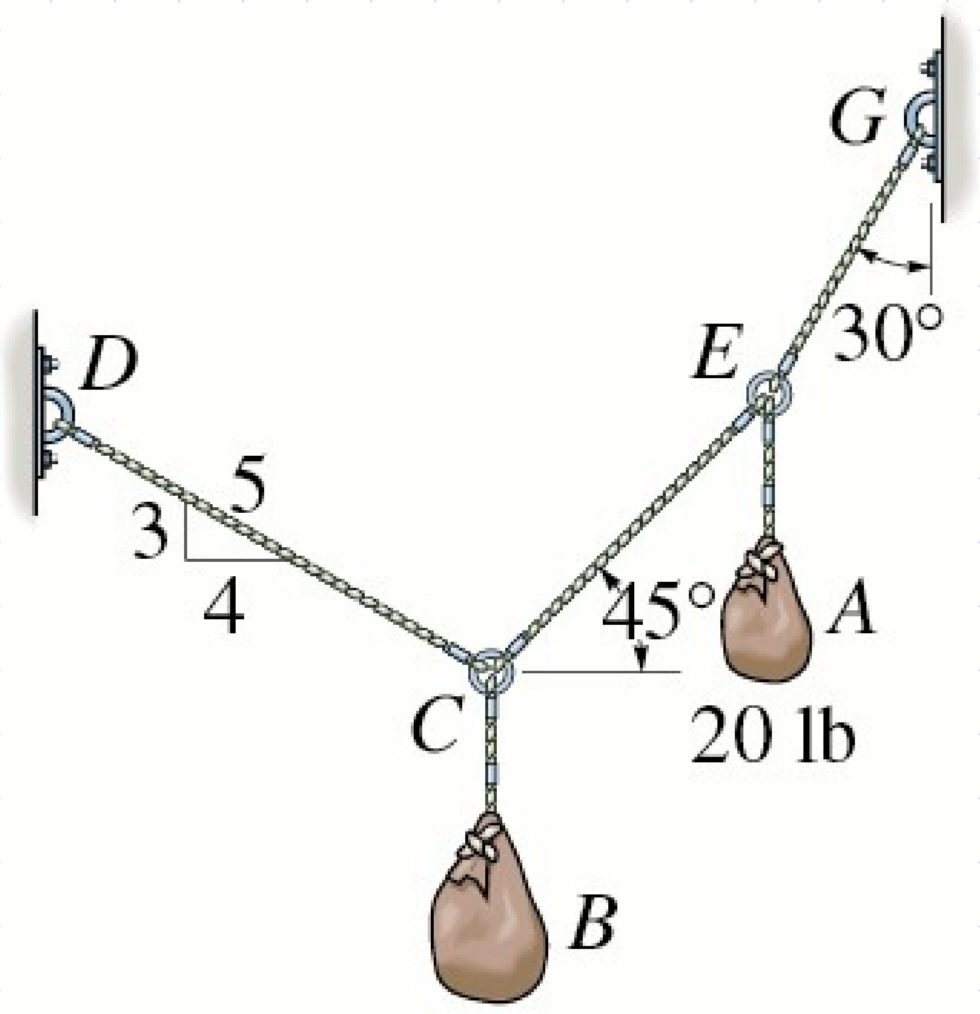


Not a Free Body Diagram!

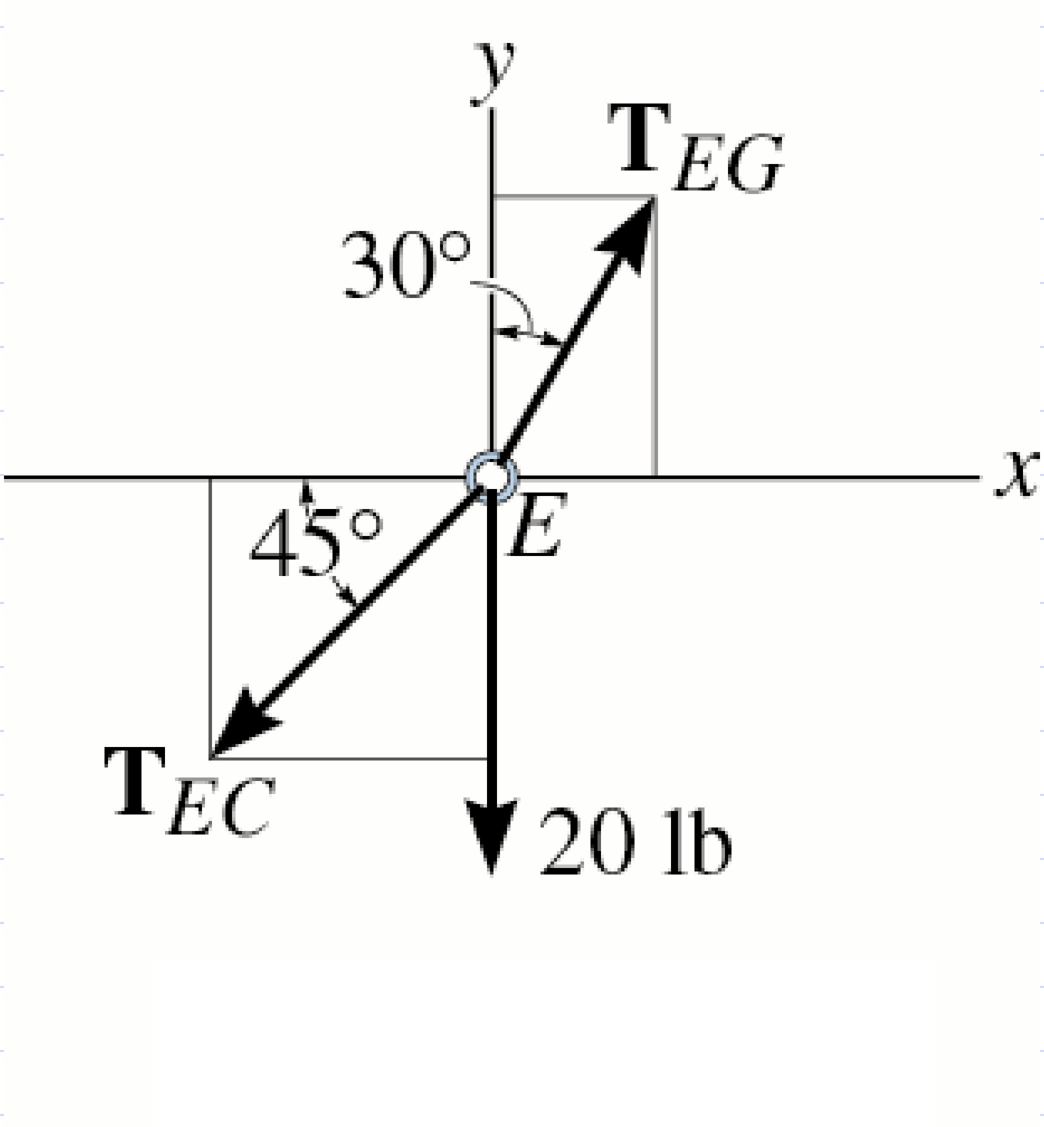
FBD



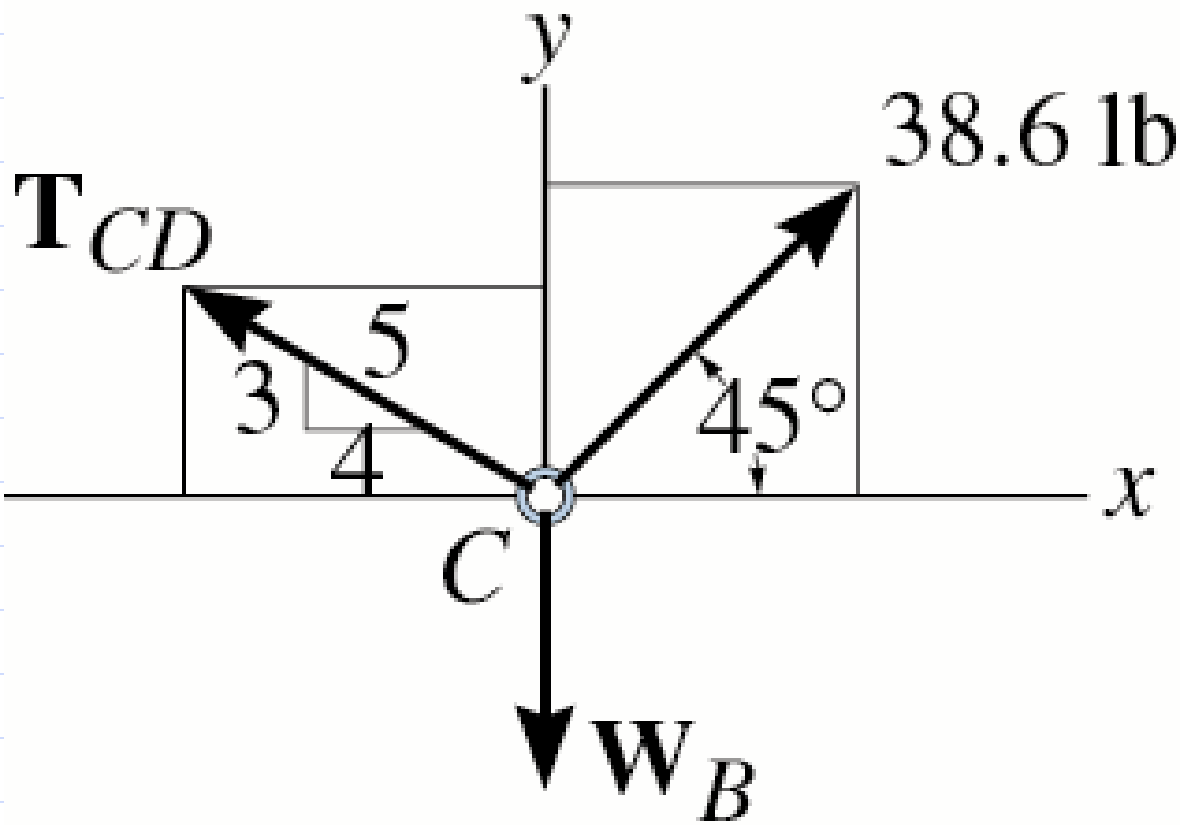
Example

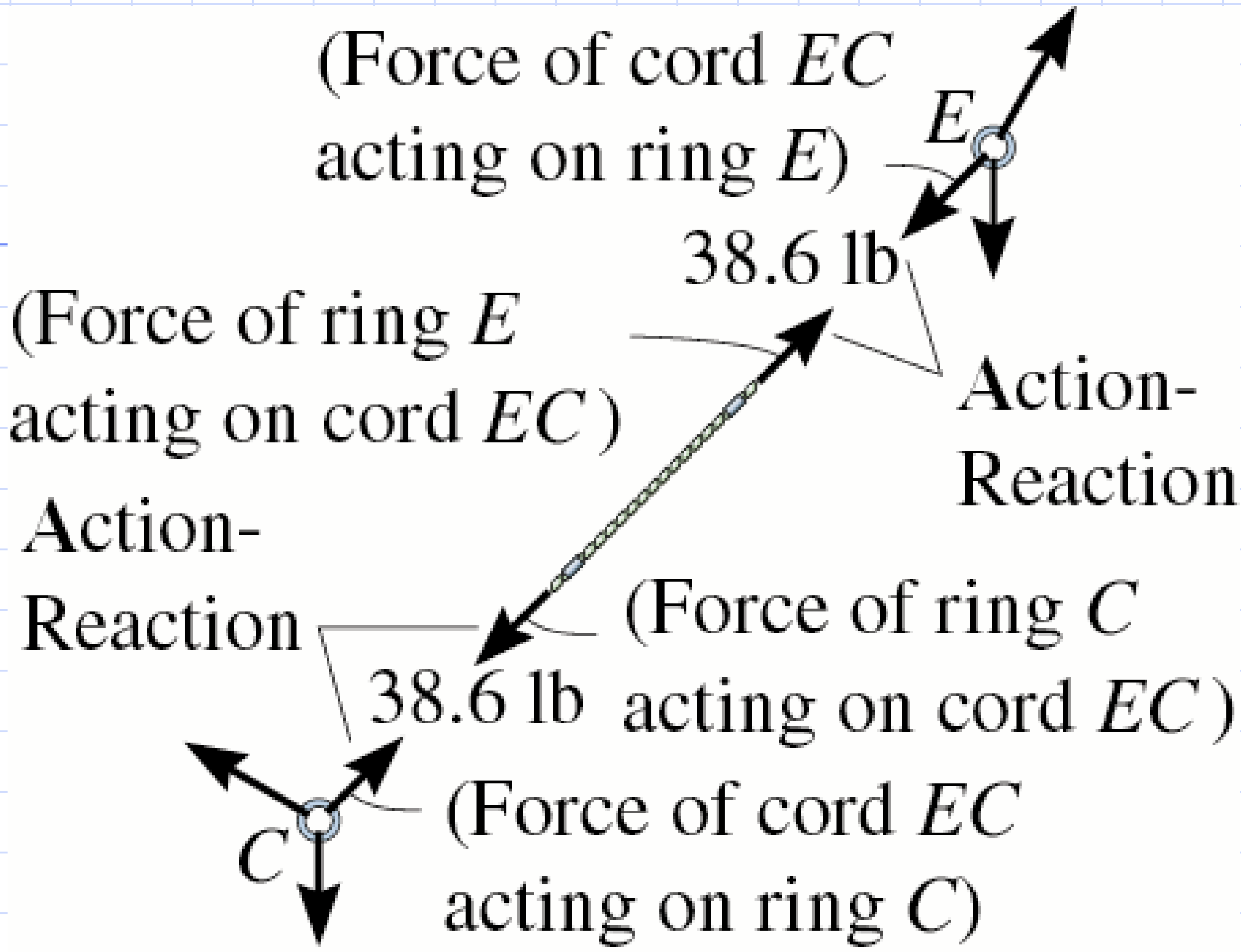


FBE of E

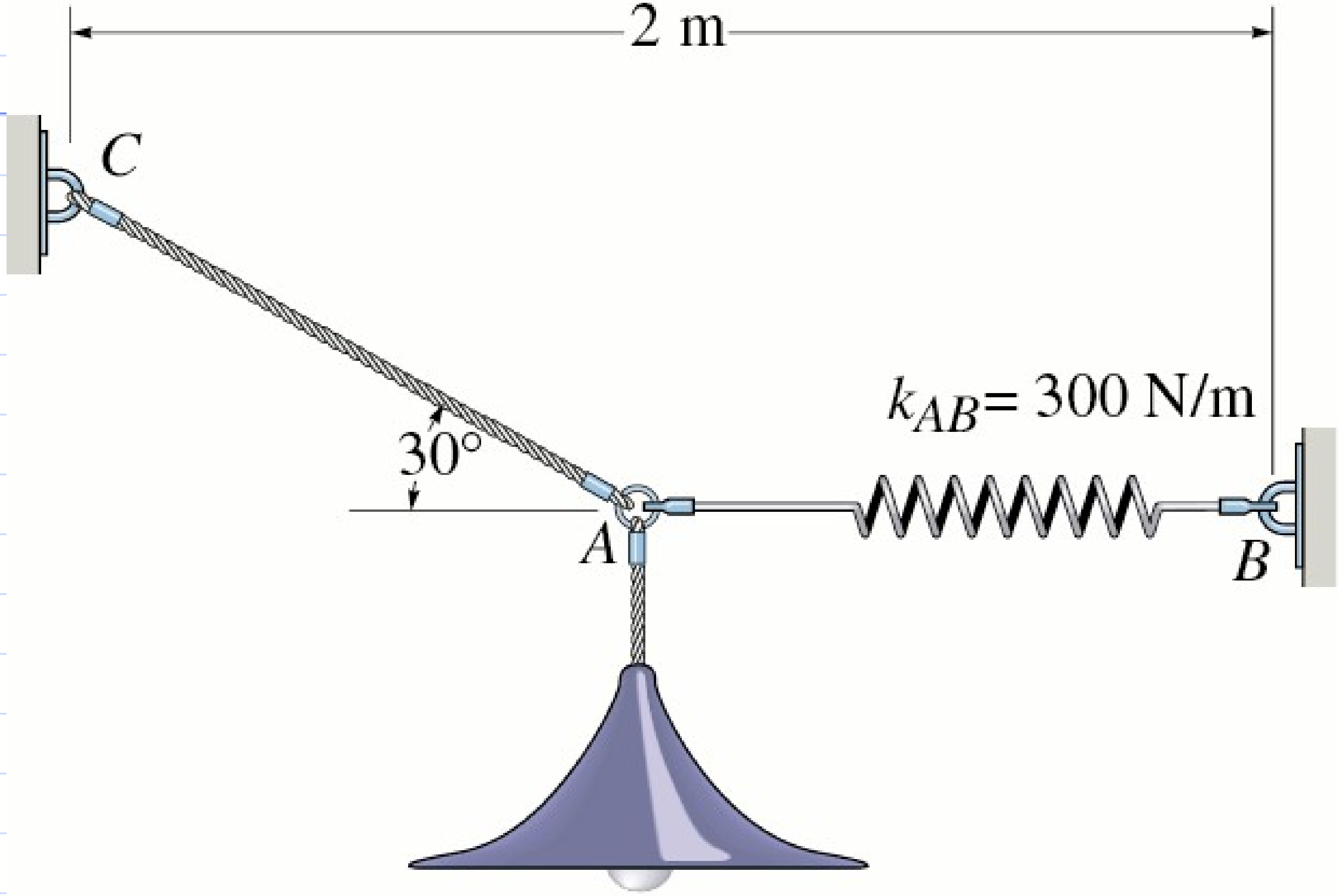


FBD of C

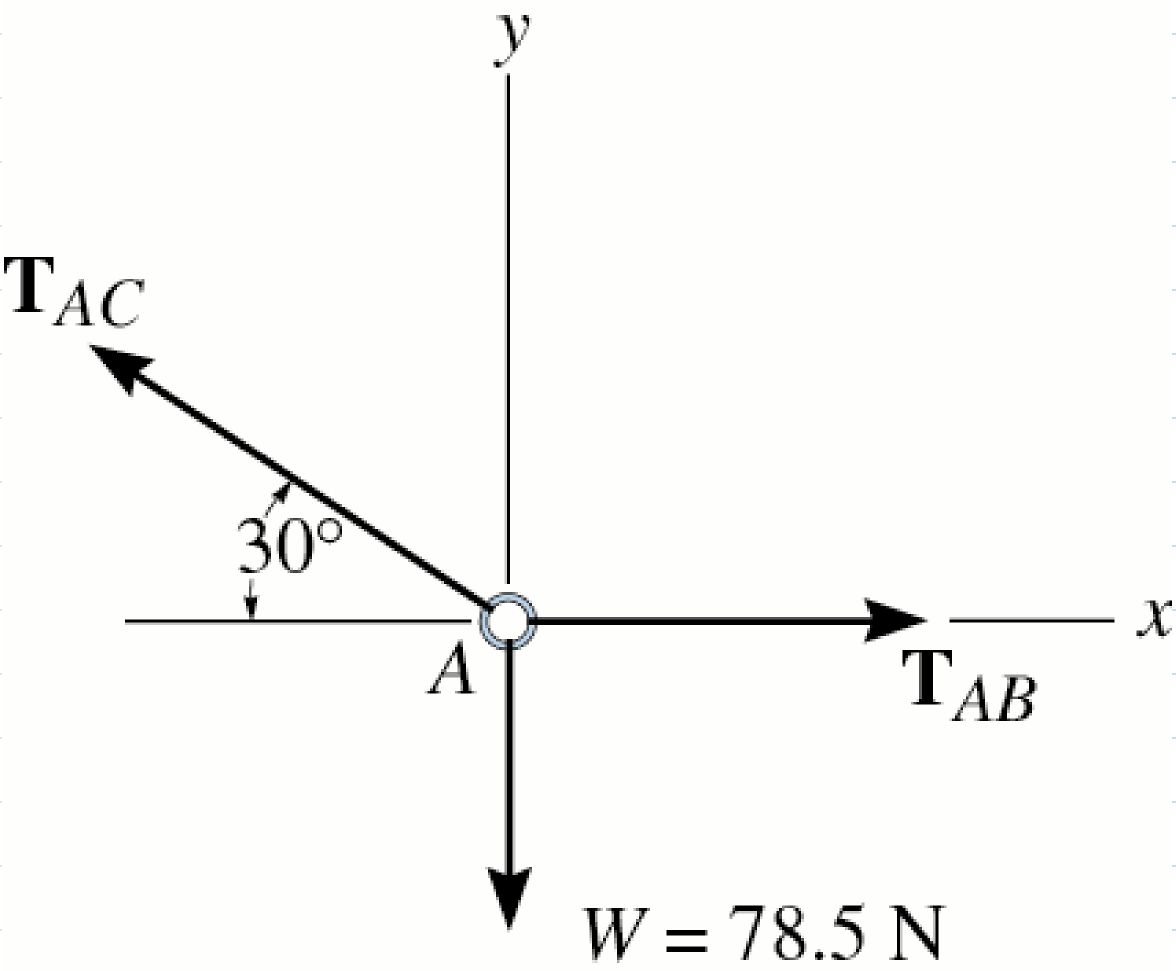



(Force of cord *EC* acting on ring *E*)
E
38.6 lb
(Force of ring *E* acting on cord *EC*)
Action-Reaction
Action-Reaction
(Force of ring *C* acting on cord *EC*)
38.6 lb
(Force of cord *EC* acting on ring *C*)
C

Example



FBD of A

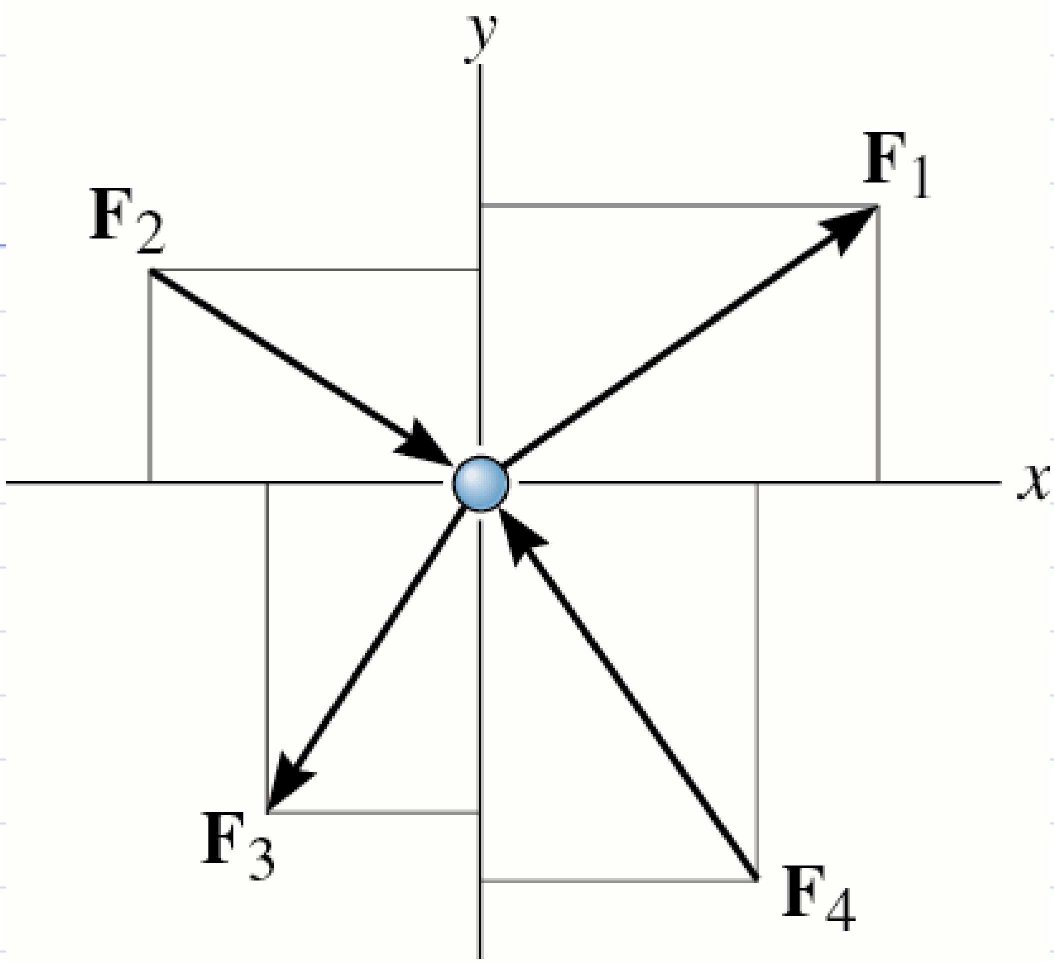


Coplanar Force System

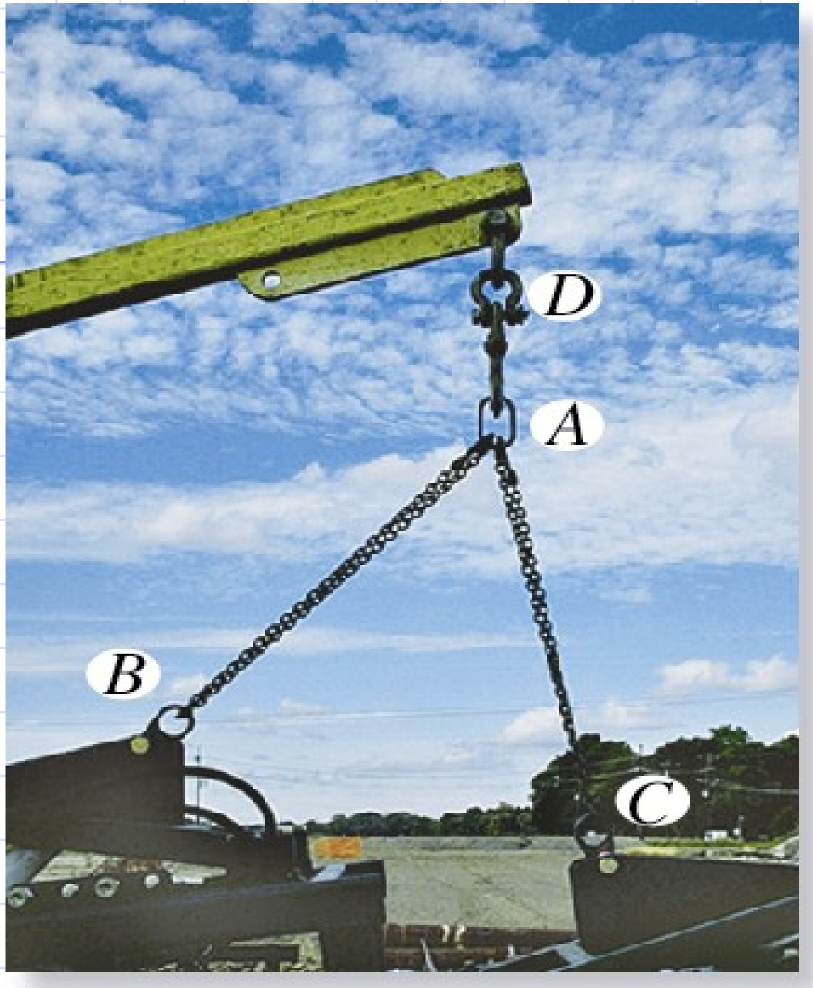
1. A two dimensional system.
2. Assumed to lie in the *x-y* plane.
3. Use **i** and **j** unit vectors.

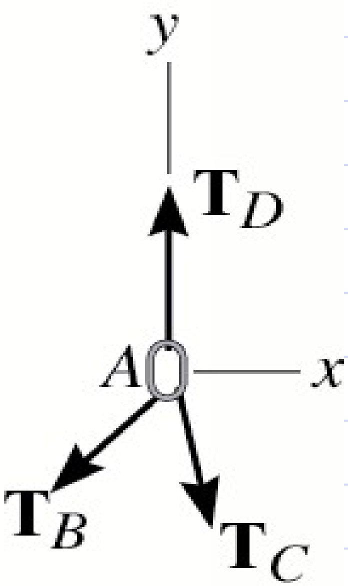
$$\sum \mathbf{F} = 0$$

$$\left(\sum F_x\right)\hat{\mathrm{i}} + \left(\sum F_y\right)\hat{\mathrm{j}} = 0$$



y
F1
F2
x
F3
F4


D
A
B
C


y
$\mathbf{T}_D$
A
x
$\mathbf{T}_B$
$\mathbf{T}_C$

2D Equilibrium Equations

$$\sum F_x = 0$$

$$\sum F_y = 0$$

alar equations of equilibrium require that the gebraic sum of the x and y components of all th rces acting on a particle be equal to zero.

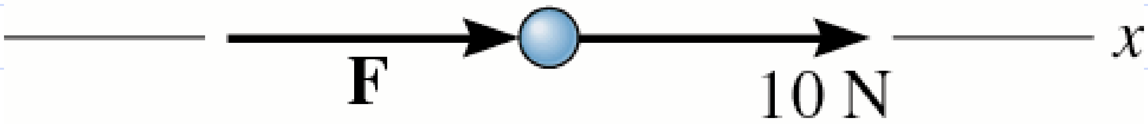
2D Equilibrium Equations

$$\sum F_x = 0$$

$$\sum F_y = 0$$

wo equations means only two unknowns can be
olved for from a single FBD.

Assume a sense for an unknown force. If the equations yield a negative value for the magnitude then the sense is opposite of what was assumed.



F + 10 N = 0
F = -10 N
F acts to the left (opposite of direction shown).

Procedure for Analysis

Free-Body Diagram

1. Establish the x, y axes in any suitable orientation.
2. Label all known and unknown force magnitudes and directions on the FBD.
3. The sense of an unknown force may be assumed.

Procedure for Analysis

Equations of Equilibrium

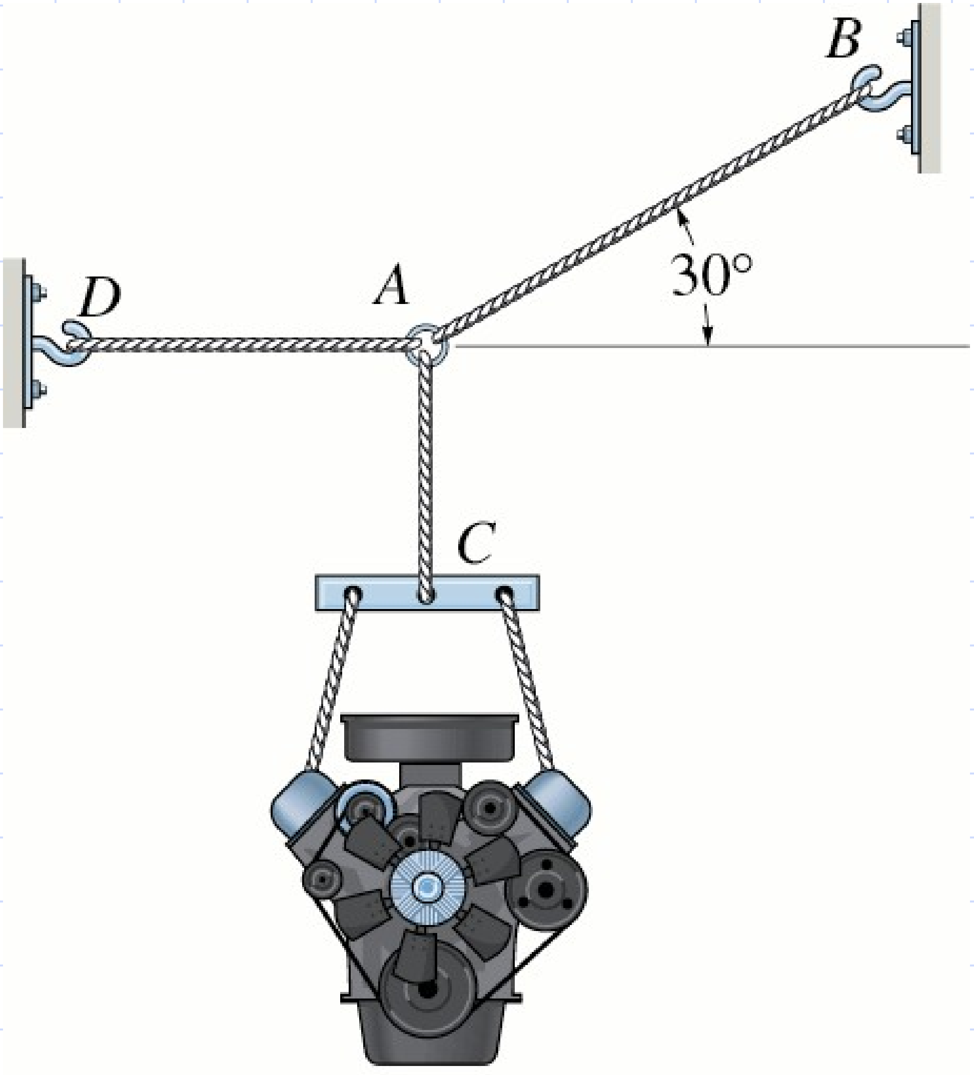
1. Apply equations of equilibrium.

$$\sum F_x = 0 \quad and \quad \sum F_y = 0$$

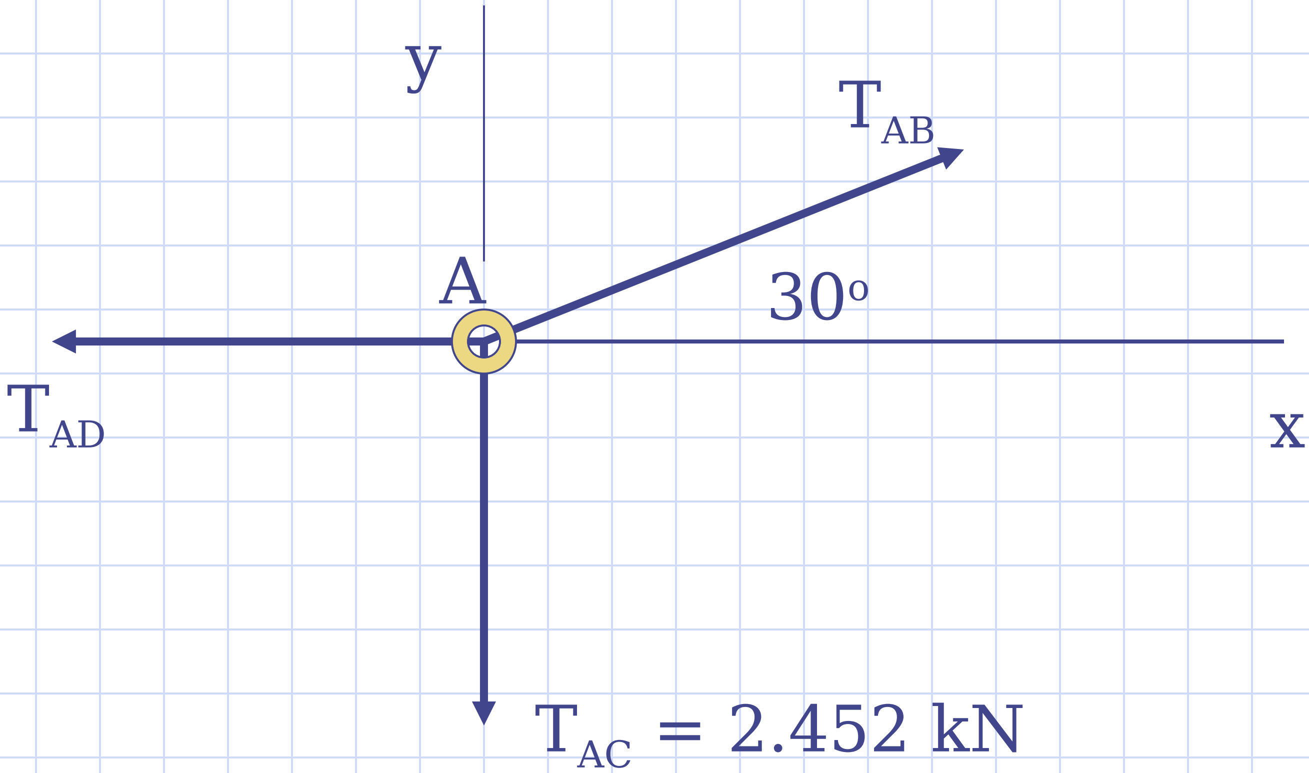
2. Components of force are positive if directed along a positive axis and negative if directed along a negative axis.
3. If solution yields a negative result the force is in the opposite sense of that shown on the FBD.

Example

Determine the tension in cables AB and AD for equilibrium of the 250 kg engine block.



solve this problem apply equilibrium equation at point A.
e weight of the object is W = 250 kg (9.81 m/s²) = 2.452 N.
s weight is supported by cable AC so TAC = 2.452 N.



Free-Body Diagram

Equilibrium Equations

$$\sum F_x = 0$$

$$T_{AB}\cos 30^o - T_{AD} = 0$$

$$\sum F_y = 0$$

$$T_{AB}\sin 30^o - 2.452\,kN = 0$$

Solving:

$$T_{AB}\sin 30^{0} - 2.452kN = 0$$

$$T_{AB}\sin 30^{0} = 2.452kN$$

$$T_{AB}(0.5000) = 2.452kN$$

$$T_{AB} = 4.904kN$$

Solving:

$$T_{AD} = T_{AB}\cos 30^{o}$$

$$T_{AD} = (4.904\ kN)(0.8660)$$

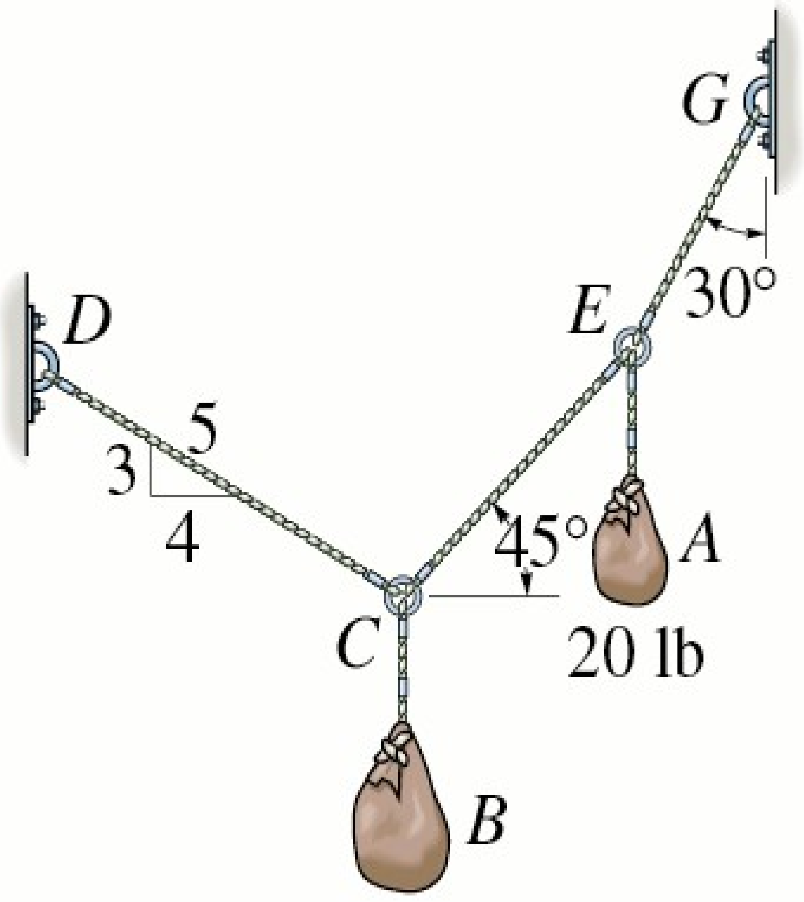
$$T_{AD} = 4.247 kN$$

Reporting our answers to three significant figures:

$$\mathbf{T_{AB} = 4.90\ kN}$$

$$\mathbf{T_{AD} = 4.25\ kN}$$

Example

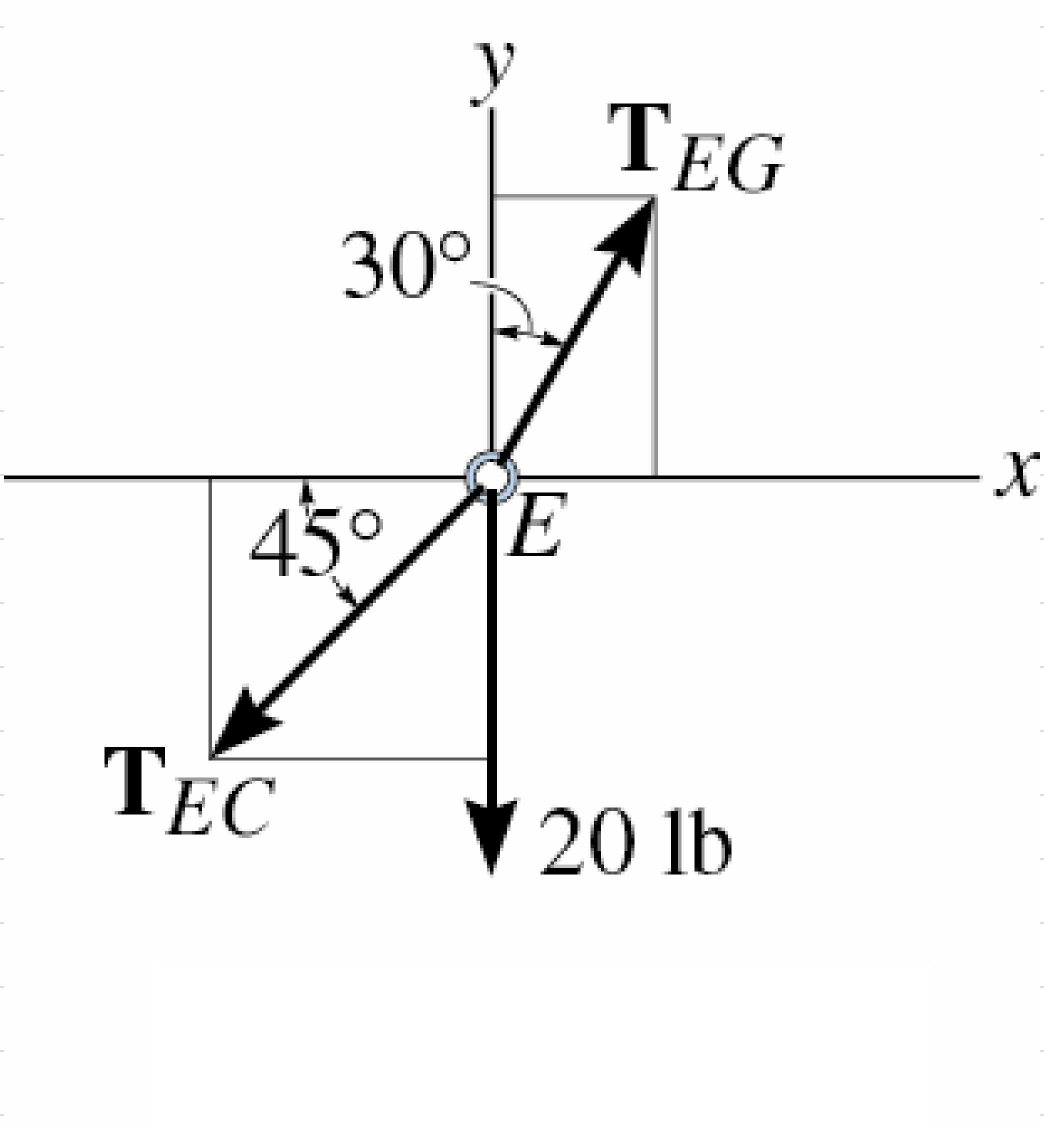


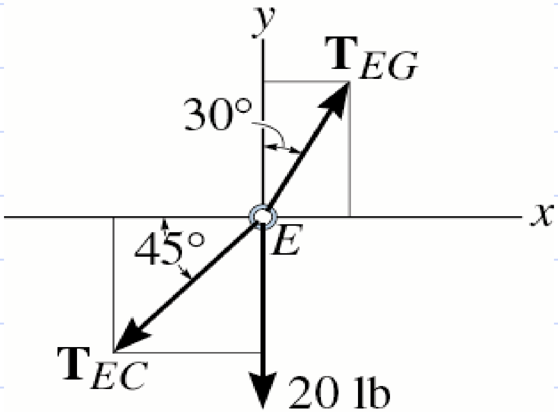
If the sack has a weight of 20 lb, determine the weight of the sack at B and the force in each cord needed to hold the system in the equilibrium position shown.

Note: there are four unknowns, the tension in the three cords and the weight B. We can draw free-body diagrams of points E and C.

Each FBD yields two equilibrium equations. Thus, we will have four equations to solve for out four unknowns.

FBE of E



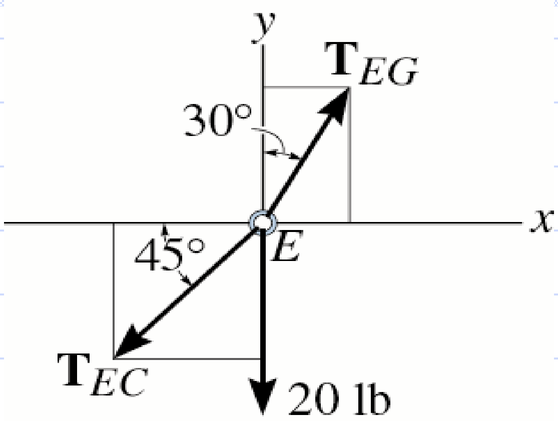


$$\sum F_x = 0$$

$$T_{EG}\sin 30^o - T_{EC}\cos 45^o = 0$$

$$T_{EG}(0.5000) - T_{EC}(0.7071) = 0$$

$$T_{EG} = 1.4142\ T_{EC}$$



$$\sum F_y = 0$$

$$T_{EG}\cos 30^o - T_{EC}\sin 45^o - 20lb = 0$$

$$T_{EG}(0.8660) - T_{EC}(0.7071) - 20lb = 0$$

$$(1.4142)(T_{EC})(0.8660) - T_{EC}(0.7071) - 20lb = 0$$

$$0.5176T_{EC} = 20lb$$

Solution

$$\sum F_x = 0$$

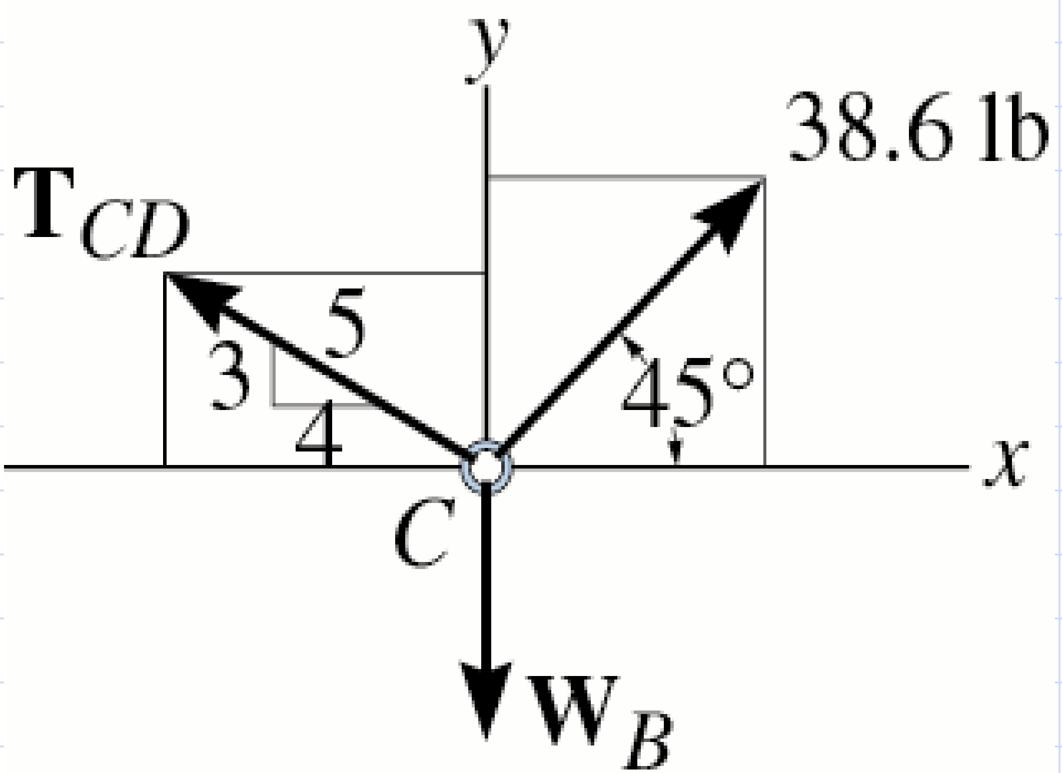
$$T_{EG} = 1.4142\ T_{EC}$$

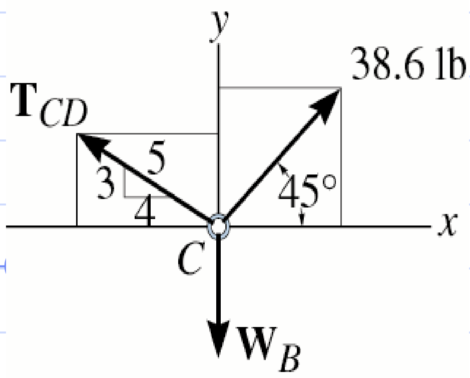
$$\sum F_y = 0$$

$$0.5176 T_{EC} = 20\ \text{lb}$$

$$\boxed{\begin{aligned} T_{EC} &= 38.6\ \text{lb} \\ T_{EG} &= 54.6\ \text{lb} \end{aligned}}$$

FBD of C



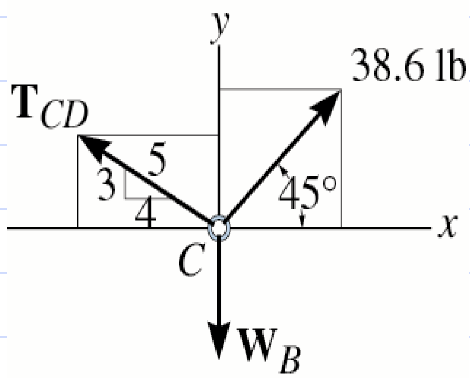


$$\sum F_x = 0$$

$$T_{CE}\cos 45^\circ - T_{CD}\left(\tfrac{4}{5}\right) = 0$$

$$38.6\ (0.7071) - T_{CD}\left(0.8000\right) = 0$$

$$T_{CD} = 34.2\ \text{lb}$$



$$\sum F_y = 0$$

$$T_{CE}\sin 45^o + T_{CD}\left(3/5\right) - W_B = 0$$

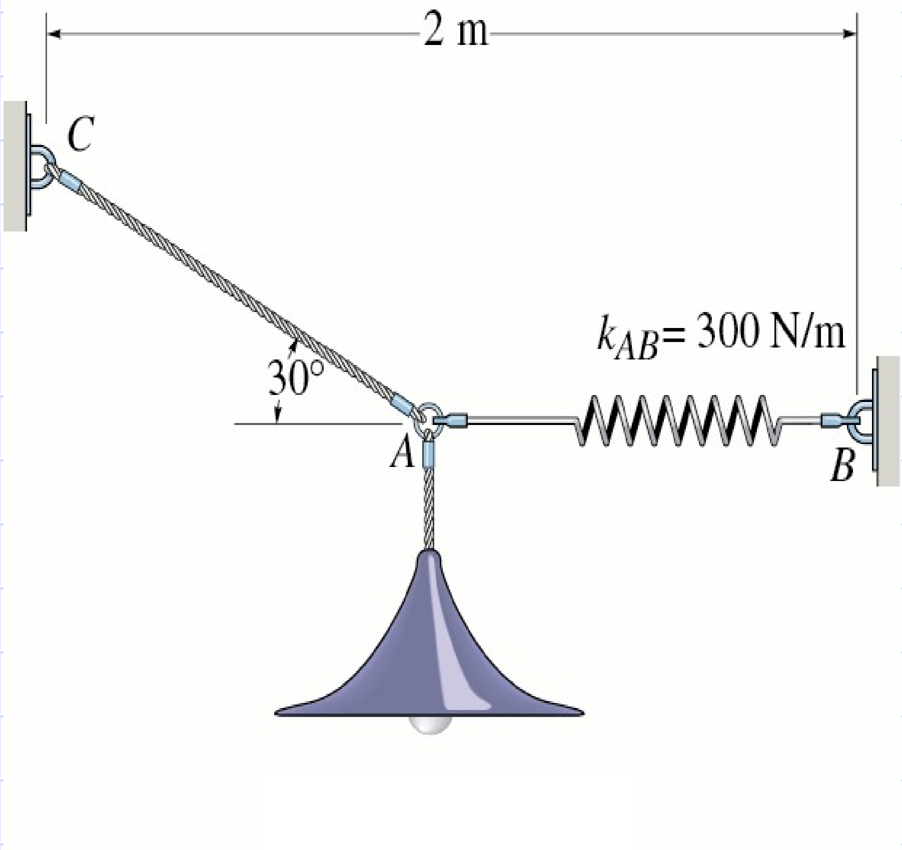
$$38.6(0.07071) + 34.2\left(0.6000\right) - W_B = 0$$

$$W_B = 47.8 \text{ lb}$$

Answers

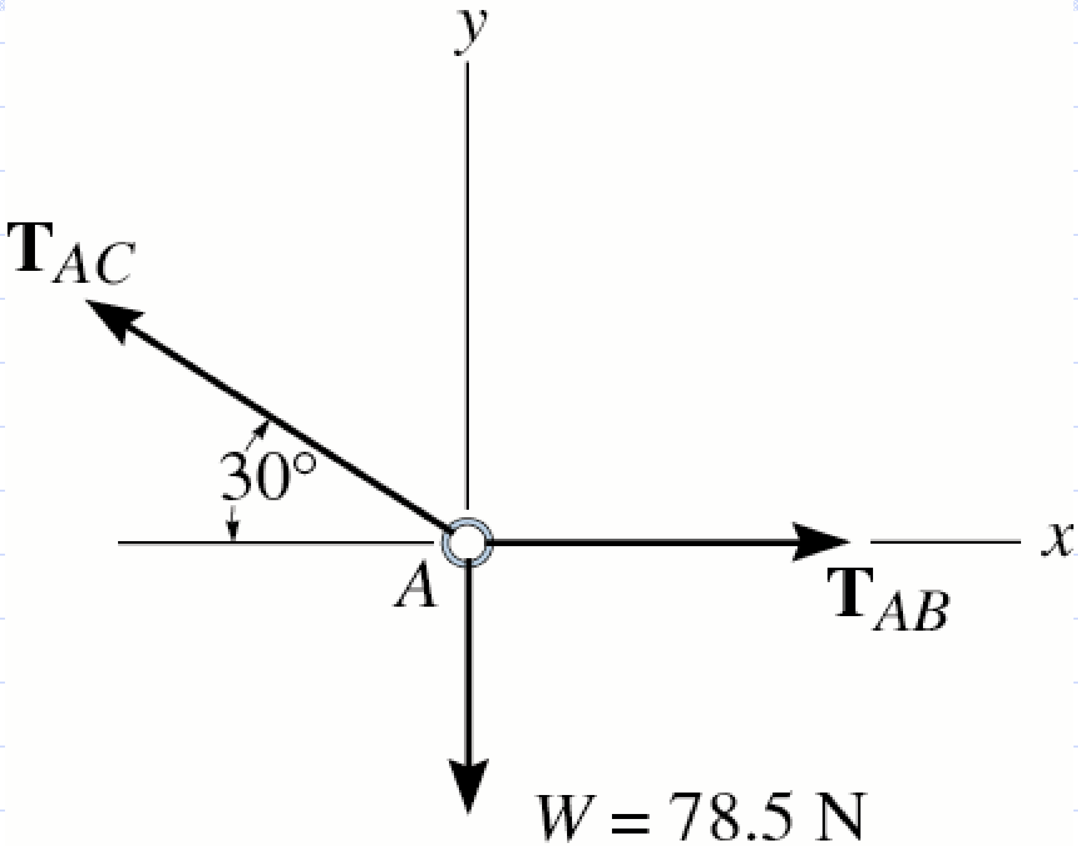
$$\boxed{\begin{array}{ll} T_{EC} = 38.6\ \text{lb} & T_{CD} = 34.2\ \text{lb} \\ T_{EG} = 54.6\ \text{lb} & W_B = 47.8\ \text{lb} \end{array}}$$

Example



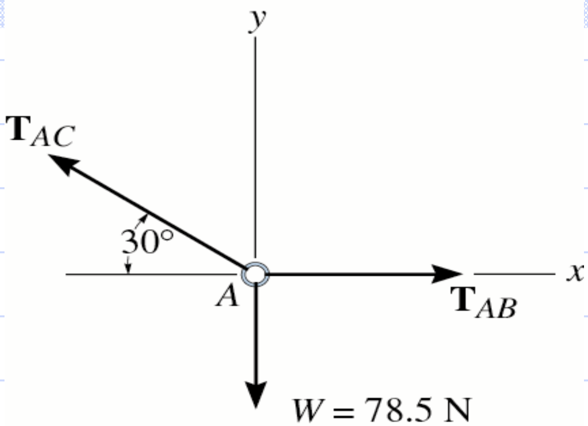
Determine the required length of cord AC so that the 8 kg lamp is suspended in the position shown. The undeformed length of spring AB is 0.4 m and the spring has a stiffness of 300 N/m

FBD of A



$$\mathbf{W} = \left(9.81\frac{\mathbf{m}}{\mathbf{s}^2}\right)(8\mathbf{kg}) = 78.5\mathbf{N}$$

Equilibrium



$$\sum F_x = 0 \Rightarrow T_{AB} - T_{AC}\cos 30^o = 0$$

$$\sum F_y = 0 \Rightarrow T_{AC}\sin 30^o - 78.5N = 0$$

$$T_{AC} = 157.0N$$

$$T_{AB} = 136.0N$$

Spring

$$T_{AB} = 136.0\,N$$

$$T_{AB} = k_{AB}\, s_{AB}$$

$$136.0\,N = 300\frac{N}{m}\, s_{AB}$$

$$s_{AB} = 0.453\,m$$

Stretched length:

$$L_{AB} = 0.4\,m + 0.453\,m = 0.853\,m$$

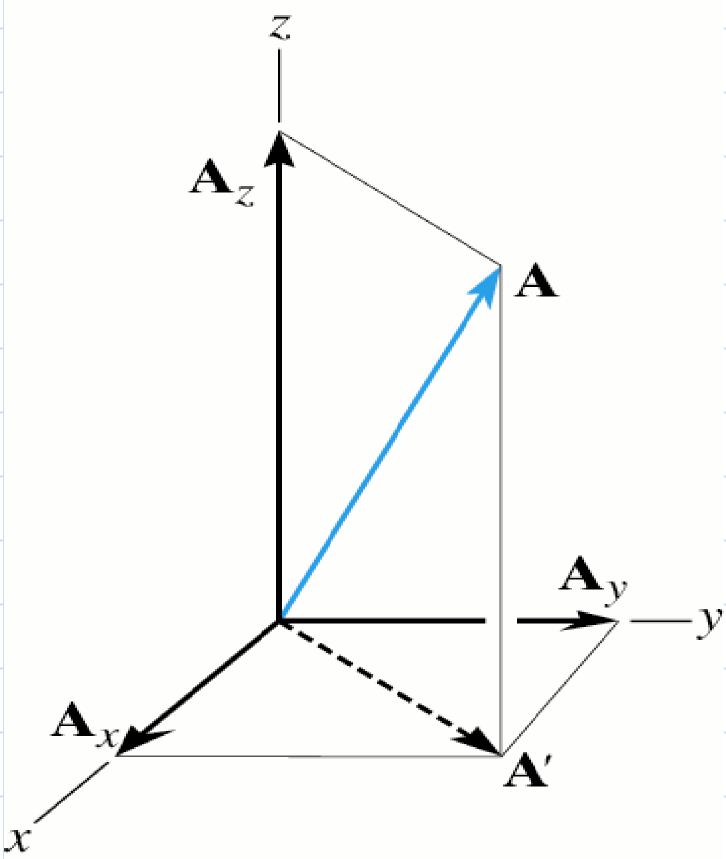
CORD CA

Horizontal Distance from C to A:

$$2\mathrm{m} = L_{AC}\cos 30^{o} + 0.853\mathrm{m}$$

$$L_{AC} = 1.32\mathrm{m}$$

Rectangular Components



$$\vec{A} = \vec{A}_x + \vec{A}_y + \vec{A}_z$$

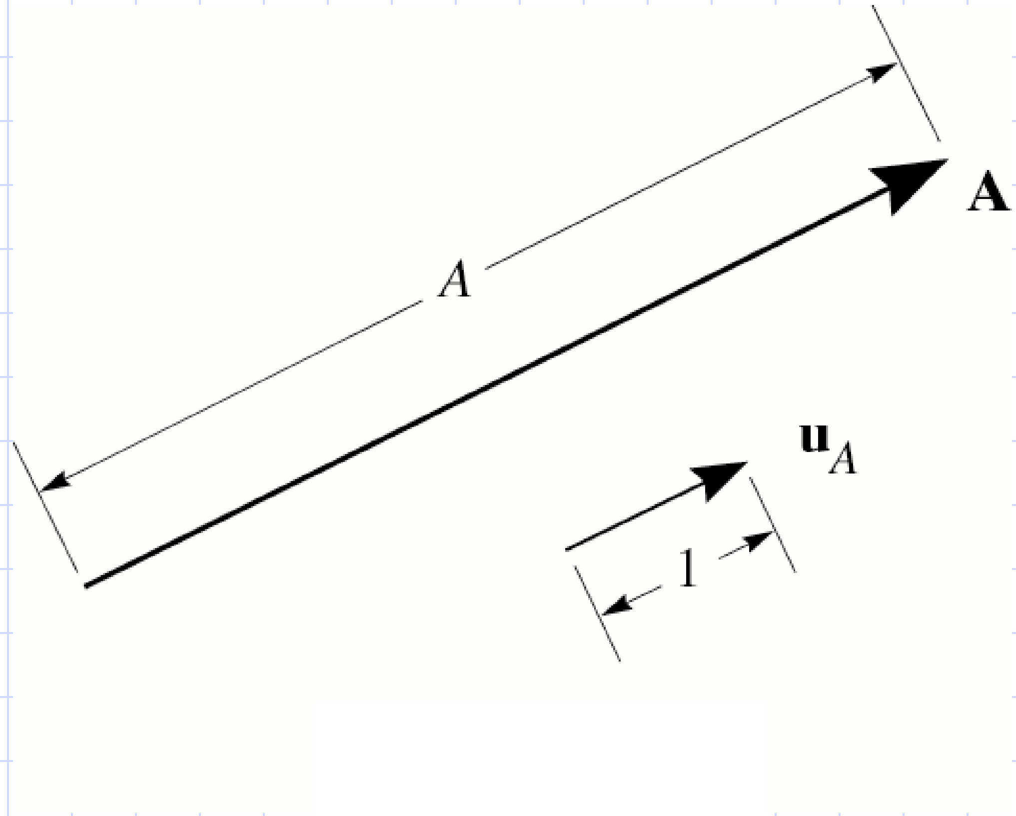
Unit Vectors

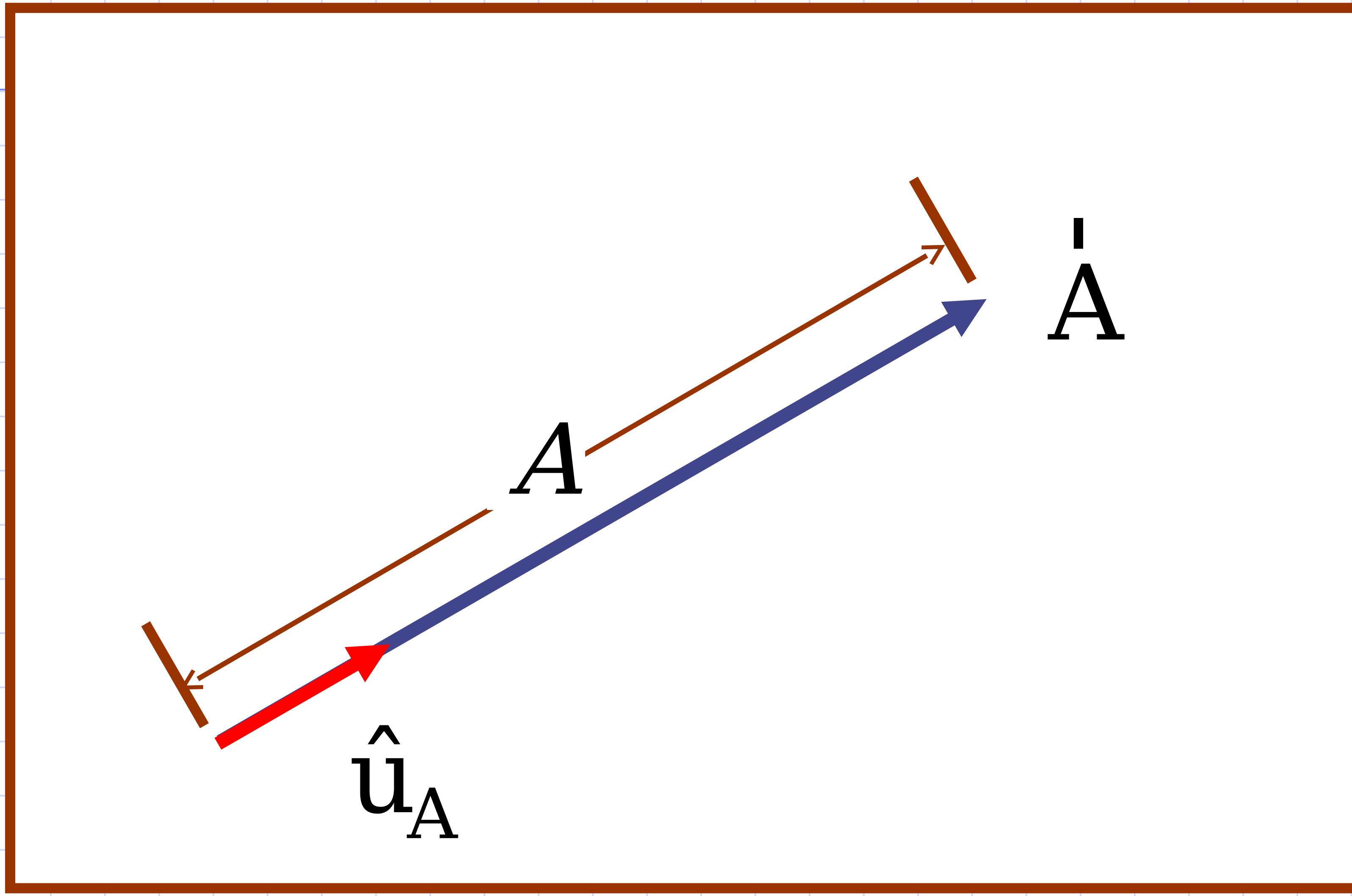
Unit Vector: a vector having magnitude of 1.

$$\hat{u}_A = \frac{\vec{A}}{A}$$

or

$$\vec{A} = A\hat{u}_A$$





A
A
û_A

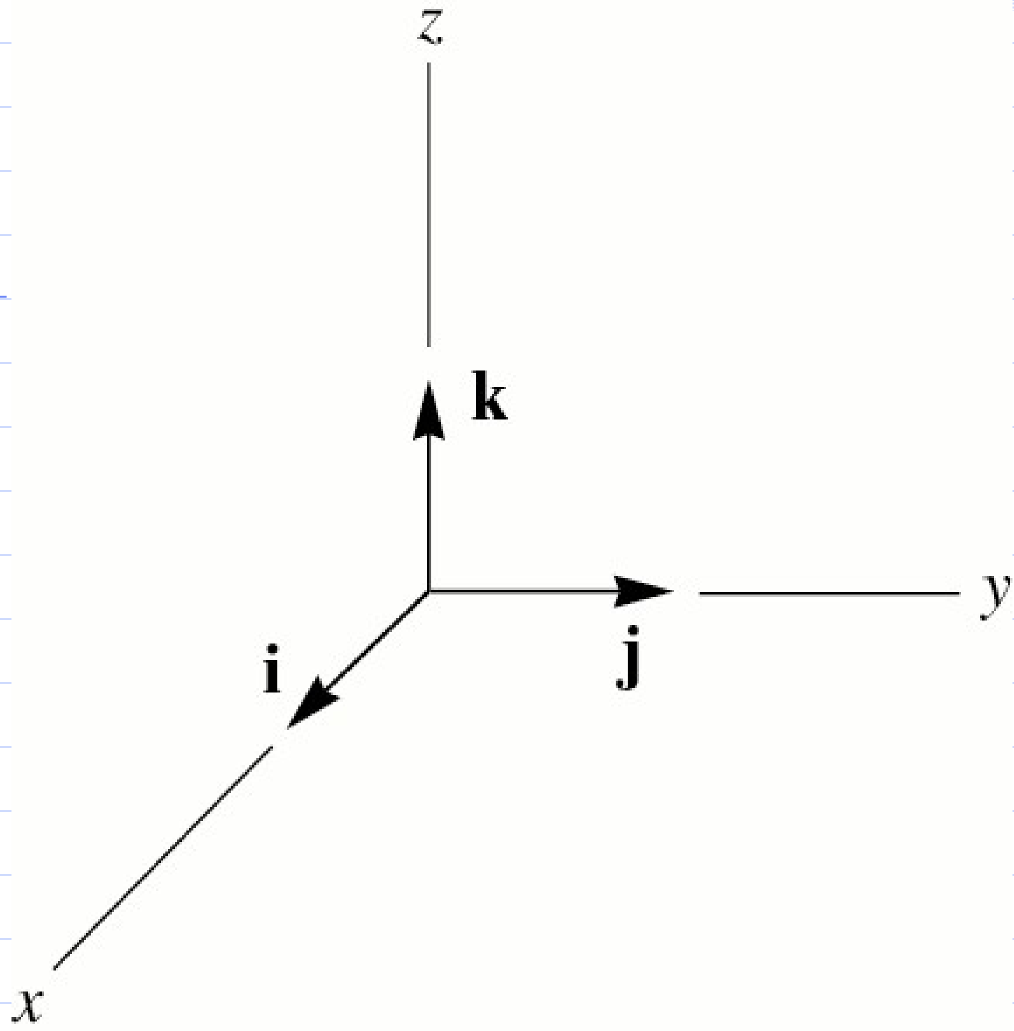
Cartesian Unit Vectors

Unit Vectors in Coordinate Directions:

$\hat{i}$ *Unit vector in the x-direction*

$\hat{j}$ *Unit vector in the y-direction*

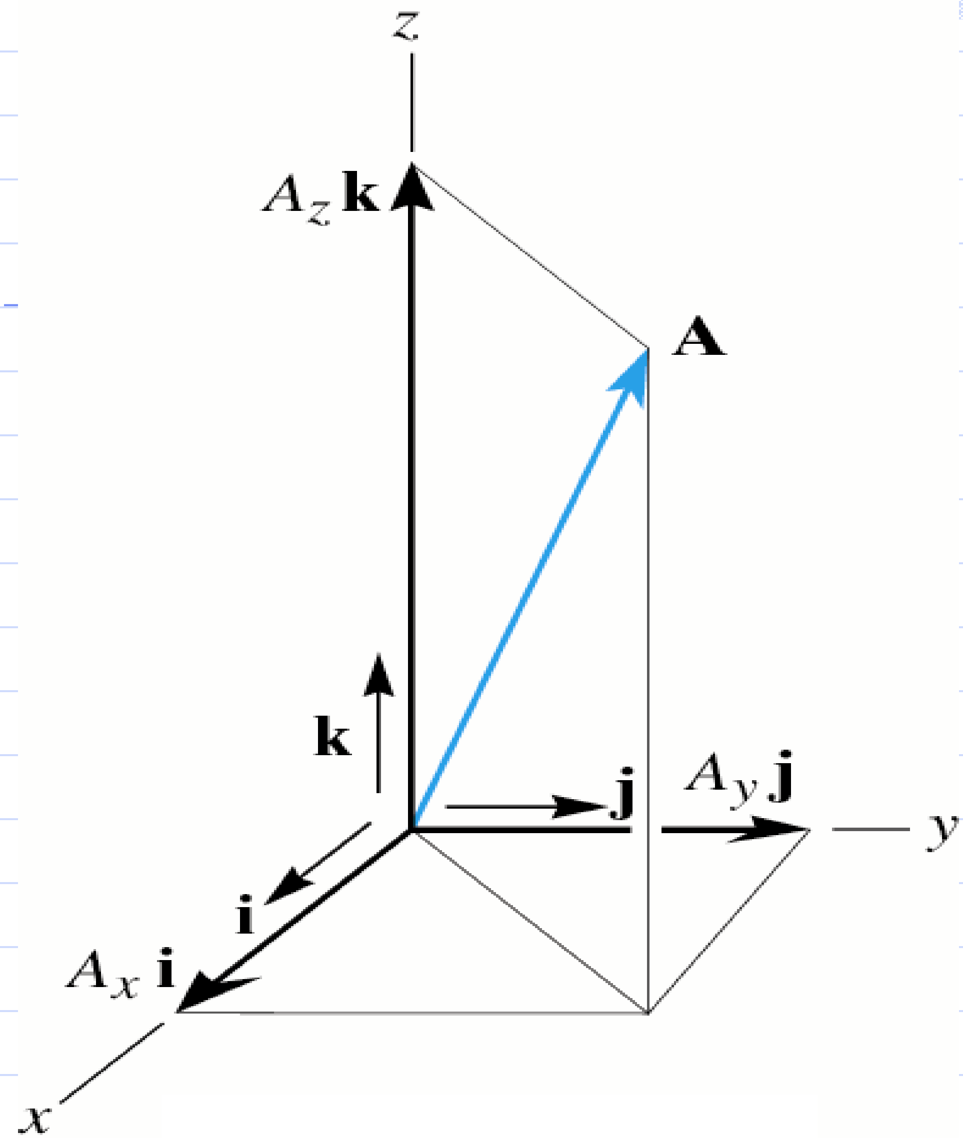
$\hat{k}$ *Unit vector in the z-direction*



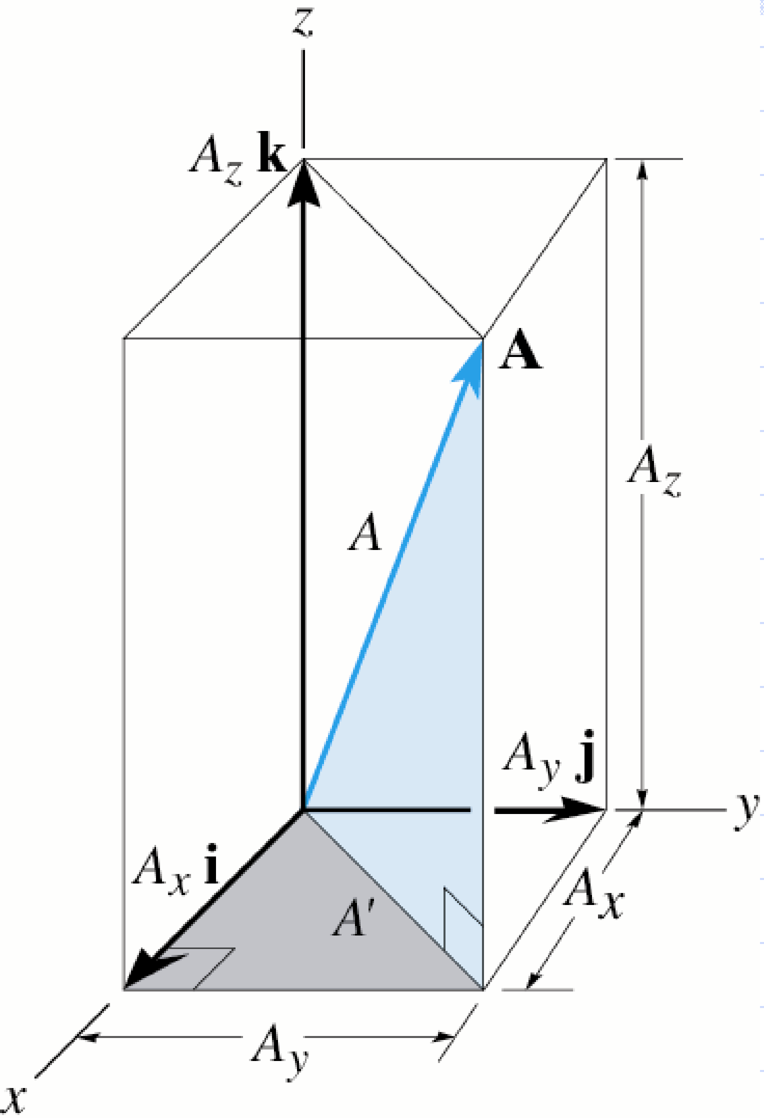
z
k
y
i
j
x

Cartesian Vector Representation

$$\vec{A} = A_x\hat{i} + A_y\hat{j} + A_z\hat{k}$$



$$\vec{A} = A_x\hat{i} + A_y\hat{j} + A_z\hat{k}$$

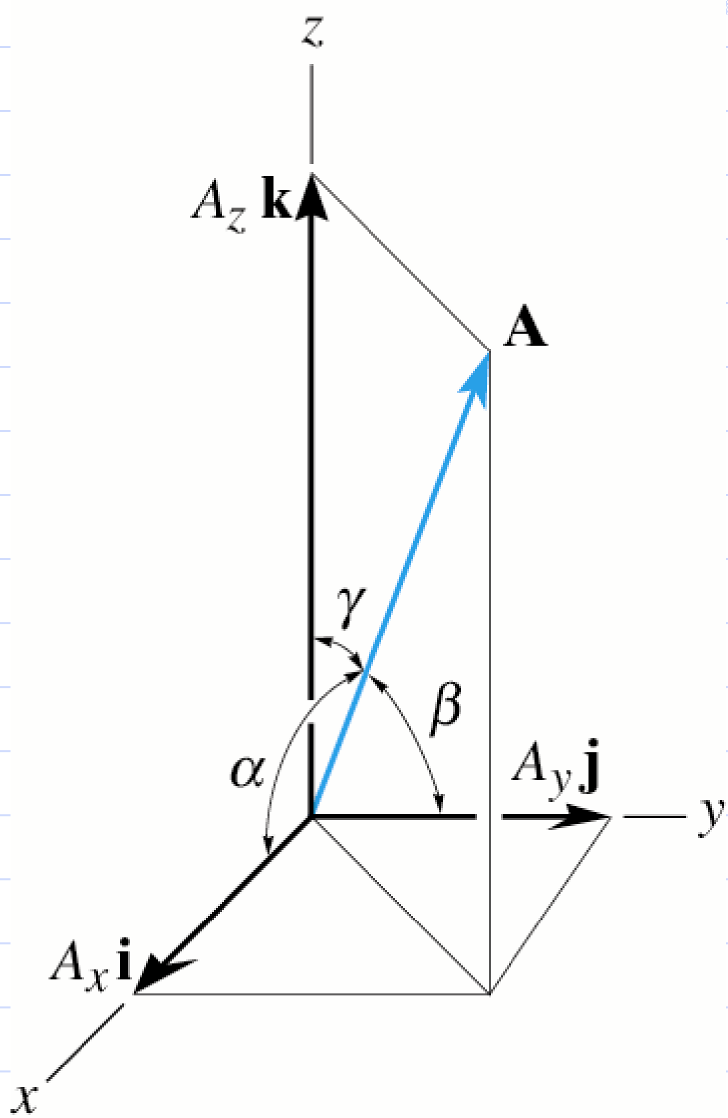


Magnitude

$$A = \sqrt{A'^2 + A_z^2}$$

$$A' = \sqrt{A_x^2 + A_y^2}$$

$$A = \sqrt{A_x^2 + A_y^2 + A_z^2}$$



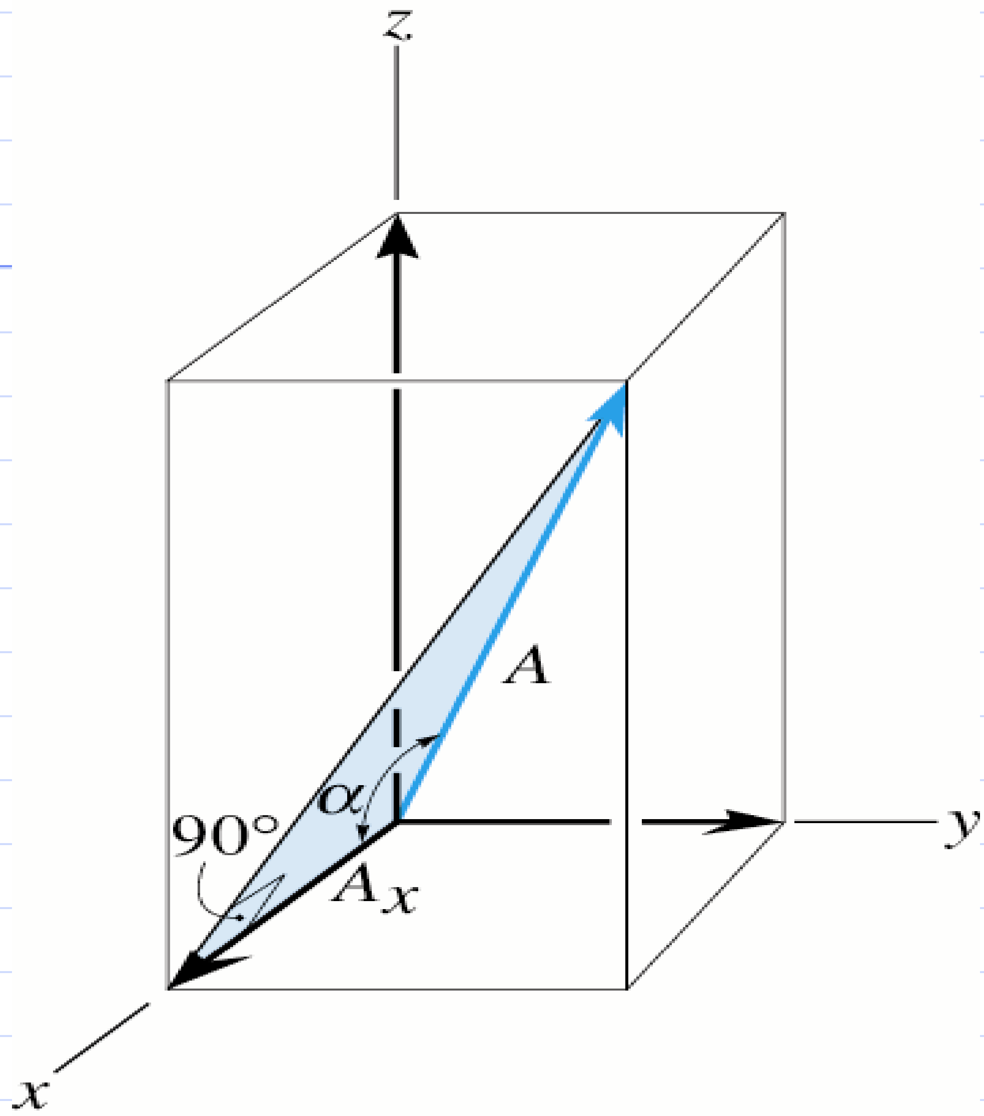
α, β, and γ are the coordinate direction angles.
These are the angles between **A** and the reference axes.
The cosines of these angels are called the direction cosines.

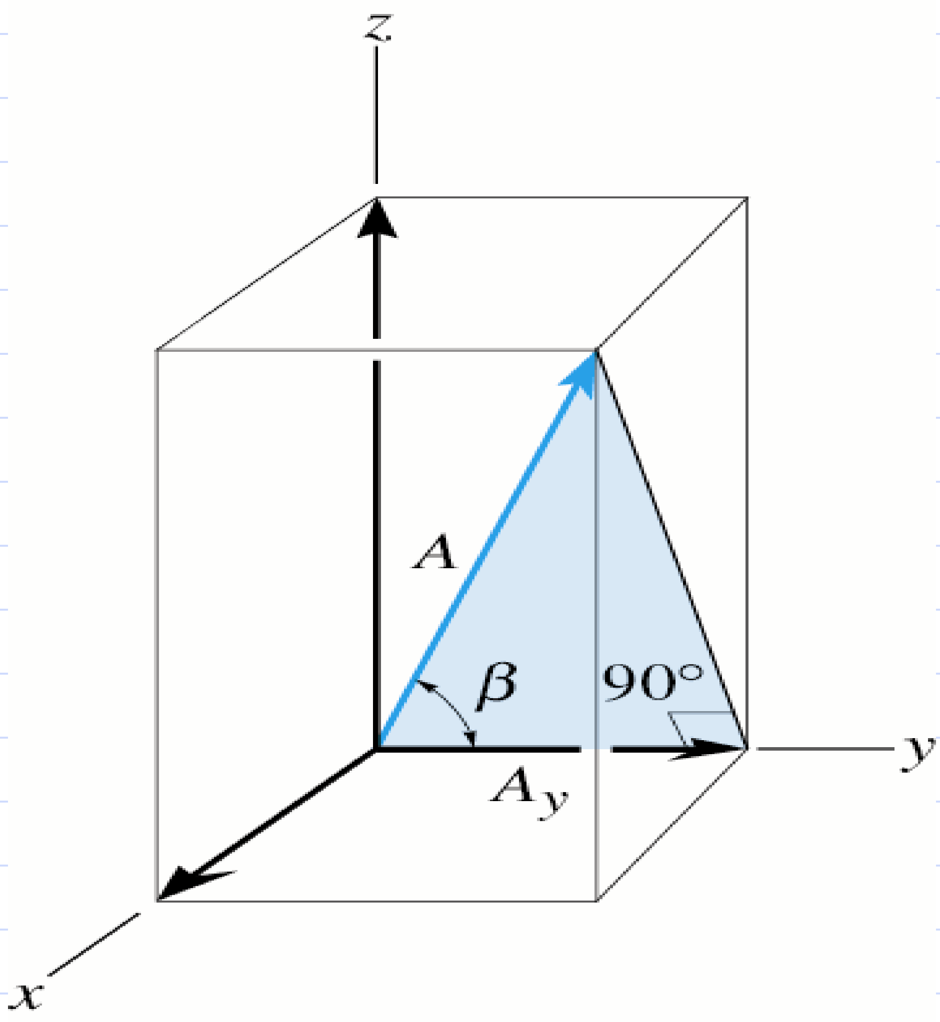
Direction Cosines

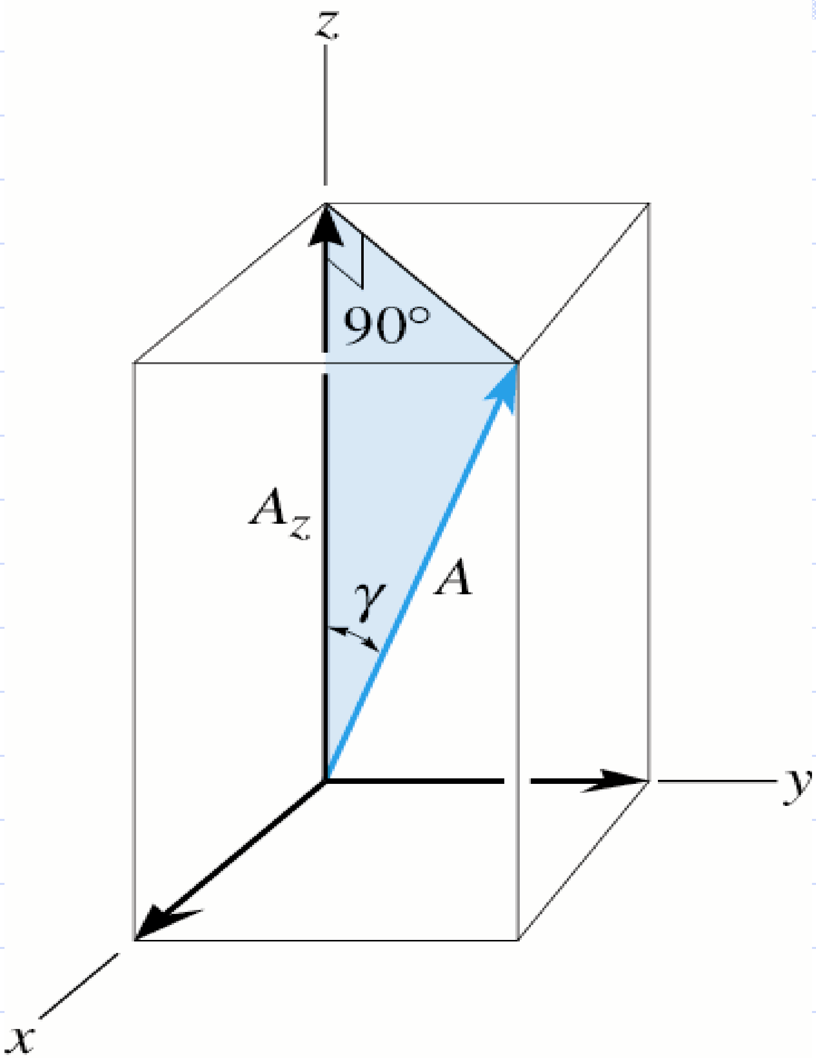
$$\cos\alpha = \frac{A_x}{A}$$

$$\cos\beta = \frac{A_y}{A}$$

$$\cos\gamma = \frac{A_z}{A}$$


z
y
x
A
α
90°
A_x


z
A
β
90°
y
A_y
x



z
90°
A_z
γ
A
y
x

$$\vec{A} = A_x\hat{i} + A_y\hat{j} + A_z\hat{k}$$

$$\hat{u}_A = \frac{\vec{A}}{A} = \frac{A_x}{A}\hat{i} + \frac{A_y}{A}\hat{j} + \frac{A_z}{A}\hat{k}$$

$$\hat{u}_A = (\cos\alpha)\,\hat{i} + (\cos\beta)\,\hat{j} + (\cos\gamma)\,\hat{k}$$

Important Relationship

$$\vec{A} = A\,\hat{u}_A$$

$$\vec{A} = A\cos\alpha\,\hat{i} + A\cos\beta\,\hat{j} + A\cos\gamma\,\hat{k}$$

$$\vec{A} = A_x\,\hat{i} + A_y\,\hat{j} + A_z\hat{k}$$

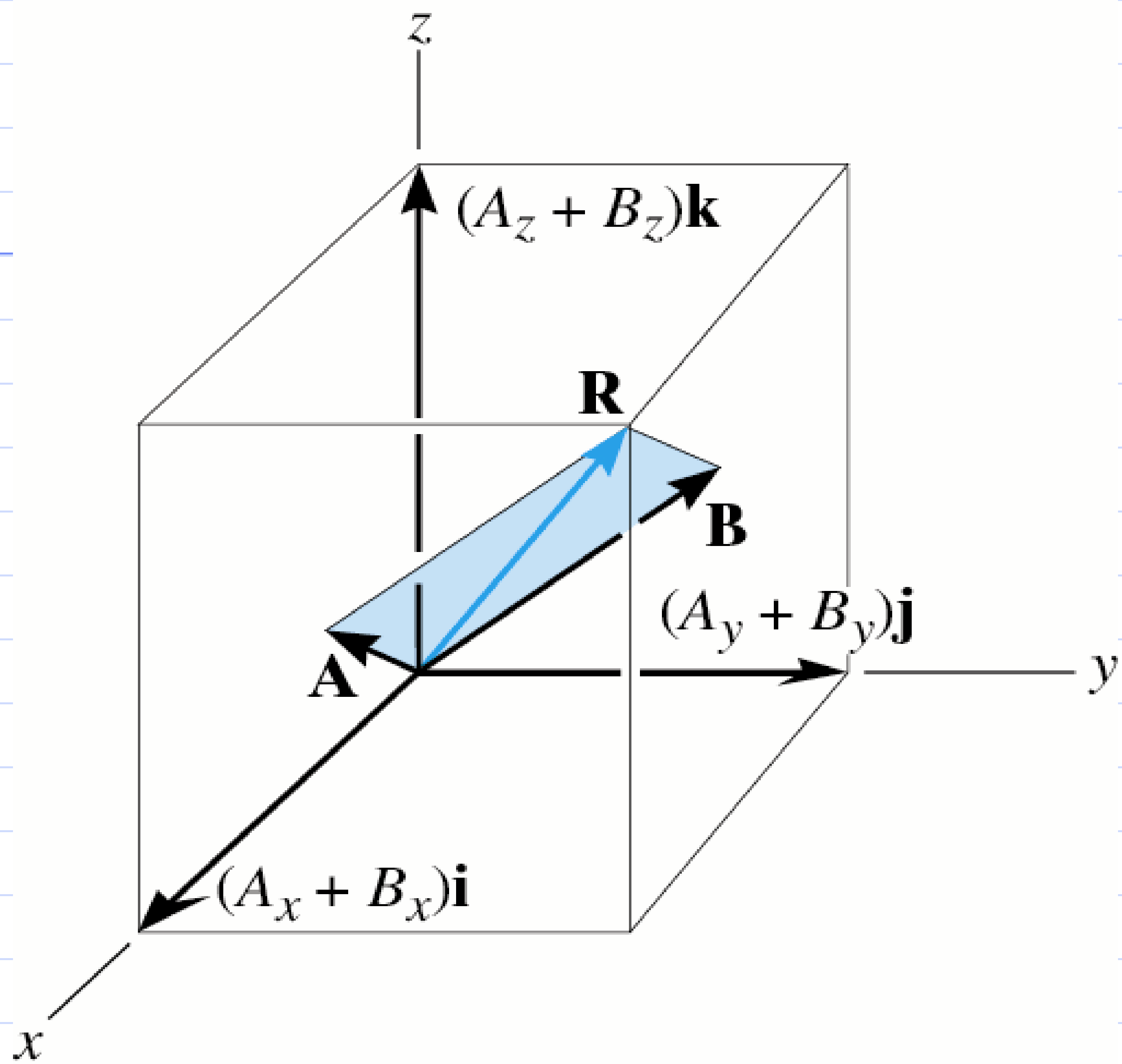
Addition and Subtraction of Cartesian Vectors

$$\vec{A} = A_x\hat{i} + A_y\hat{j} + A_z\hat{k}$$

$$\vec{B} = B_x\hat{i} + B_y\hat{j} + B_z\hat{k}$$

$$\vec{R} = \vec{A} + \vec{B}$$

$$\vec{R} = (A_x + B_x)\hat{i} + (A_y + B_y)\hat{j} + (A_z + B_z)\hat{k}$$


z
$(A_z + B_z)\mathbf{k}$
R
B
$(A_y + B_y)\mathbf{j}$
y
A
$(A_x + B_x)\mathbf{i}$
x

Addition and Subtraction of Cartesian Vectors

$$\vec{A} = A_x\hat{i} + A_y\hat{j} + A_z\hat{k}$$

$$\vec{B} = B_x\hat{i} + B_y\hat{j} + B_z\hat{k}$$

$$\vec{R}' = \vec{A} - \vec{B}$$

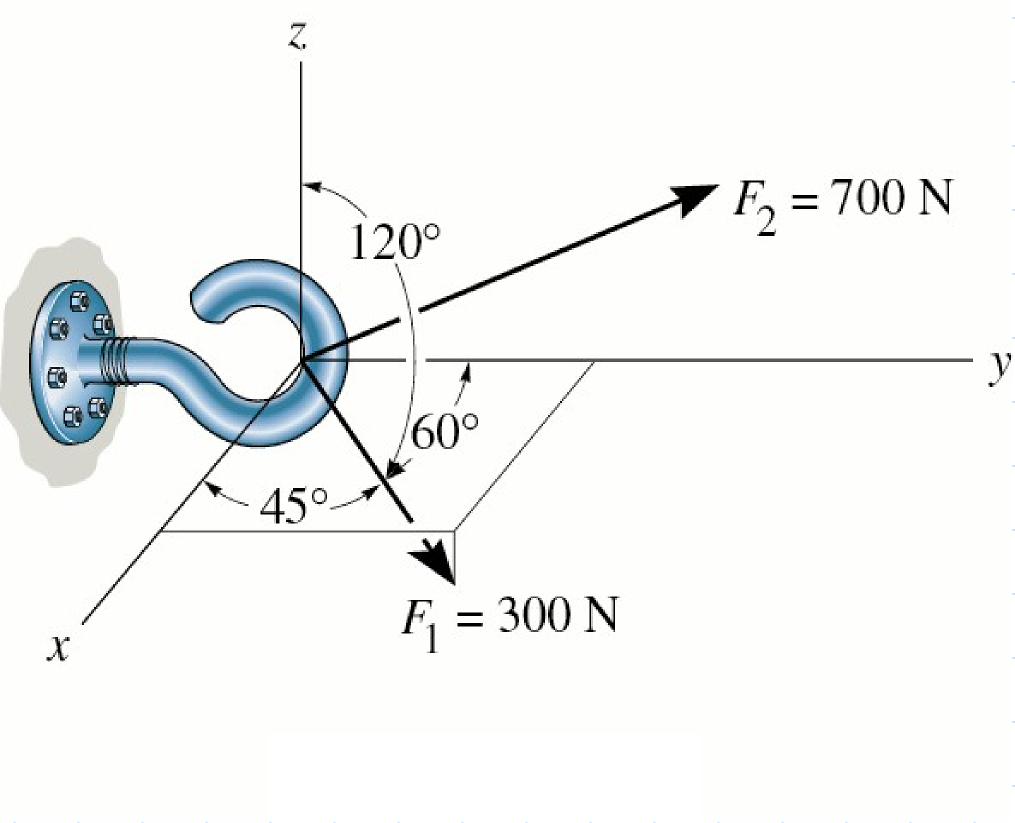
$$\vec{R}' = (A_x - B_x)\hat{i} + (A_y - B_y)\hat{j} + (A_z - B_z)\hat{k}$$

Concurrent Force Systems

A concurrent force system is one in which the lines of action of all forces intersect at a common point.

$$\vec{F}_R = \sum \vec{F} = \sum F_x \hat{i} + \sum F_y \hat{j} + \sum F_z \hat{k}$$

Example



Determine the magnitude and coordinate direction angles of the resultant

$$\vec{F}_R = [800\hat{j}] \text{ N}$$

Example

For $\vec{F}_1$:

$$\alpha_1 = 45^o \quad \beta_1 = 60^o \quad \gamma_1 = 120^o$$

$$\vec{F}_1 = F_1 \cos\alpha_1 \hat{i} + F_1 \cos\beta_1 \hat{j} + F_1 \cos\gamma_1 \hat{k}$$

$$\vec{F}_1 = (300\,N)\cos 45^o \hat{i} + (300\,N)\cos 60^o \hat{j} + (300\,N)\cos 120^o \hat{k}$$

$$\vec{F}_1 = \left[212.2\hat{i} + 150\hat{j} - 150\hat{k}\right] N$$

Example

$$\vec{F}_1 = \left[\,212.2\hat{i} + 150\hat{j} - 150\hat{k}\,\right] \text{N}$$

$$\vec{F}_2 = F_{2x}\hat{i} + F_{2y}\hat{j} + F_{2z}\hat{k}$$

$$\vec{F}_R = \left[\,800\hat{j}\,\right] \text{N}$$

Example

$$\vec{F}_R = \vec{F}_1 + \vec{F}_2$$

$$800\hat{j} = 212.2\hat{i} + 150\hat{j} - 150\hat{k} + F_{2x}\hat{i} + F_{2y}\hat{j} + F_{2z1}\hat{k}$$

$$800\hat{j} = \left(212.2 + F_{2x}\right)\hat{i} + \left(150 + F_{2y}\right)\hat{j} + \left(-150 + F_{2z}\right)\hat{k}$$

$$F_{Rx} = 212.2 + F_{2x} = 0 \quad \Rightarrow \quad F_{2x} = -212.2\mathrm{N}$$

$$F_{Ry} = 150 + F_{2y} = 800 \quad \Rightarrow \quad F_{2y} = 650\mathrm{N}$$

$$F_{Ry} = -150 + F_{2z} = 0 \quad \Rightarrow \quad F_{2z} = 150\mathrm{N}$$

Example

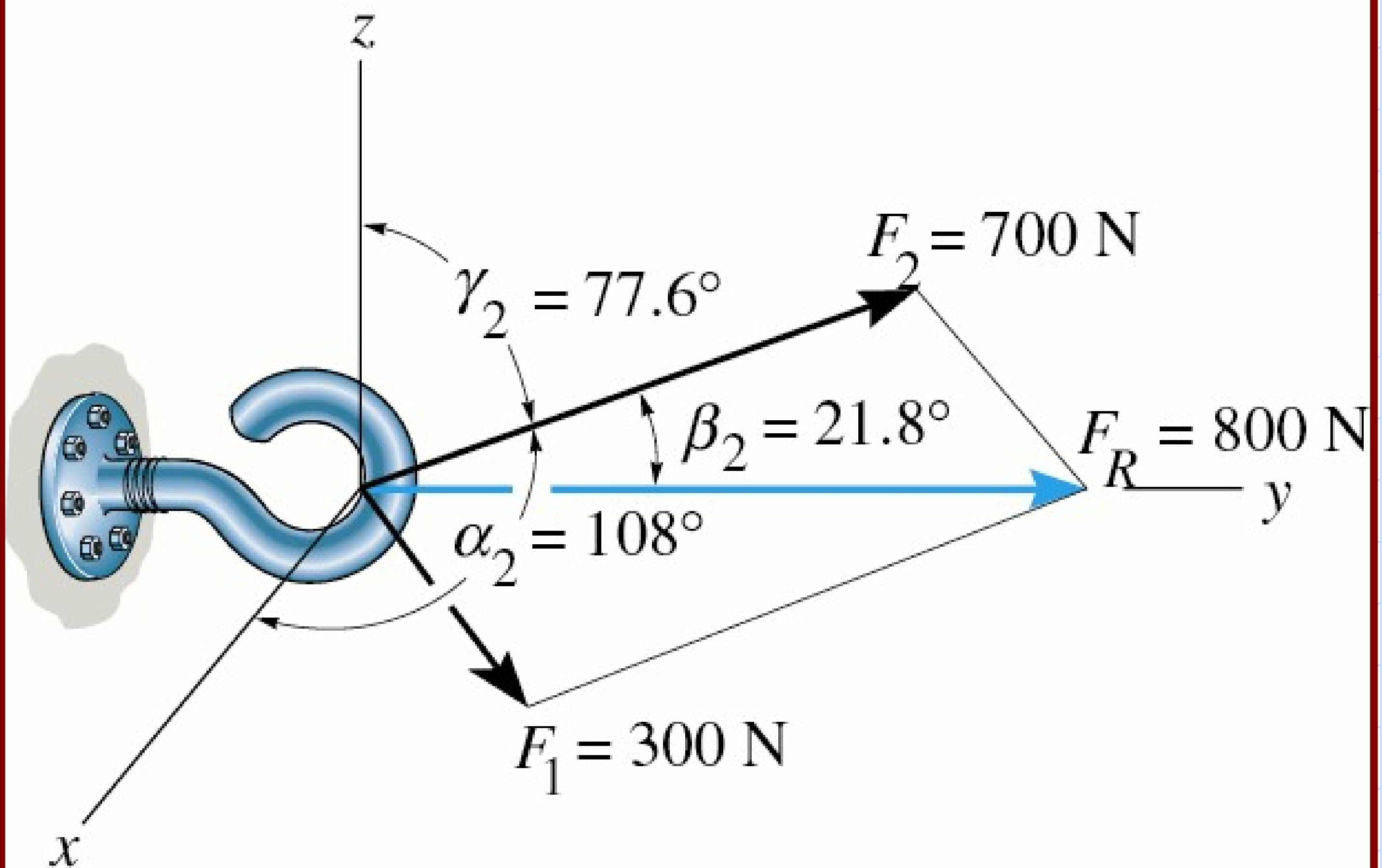
$$\vec{F}_2 = \left(-212.2\hat{i} + 650\hat{j} + 150\hat{k}\right) \text{N}$$

$$700 = \sqrt{(-212.2)^2 + (650)^2 + (150)^2}$$

$$\cos\alpha_2 = \frac{-212.2}{700} \quad \Rightarrow \quad \alpha_2 = \cos^{-1}\left(\frac{-212.2}{700}\right) = 108^{\circ}$$

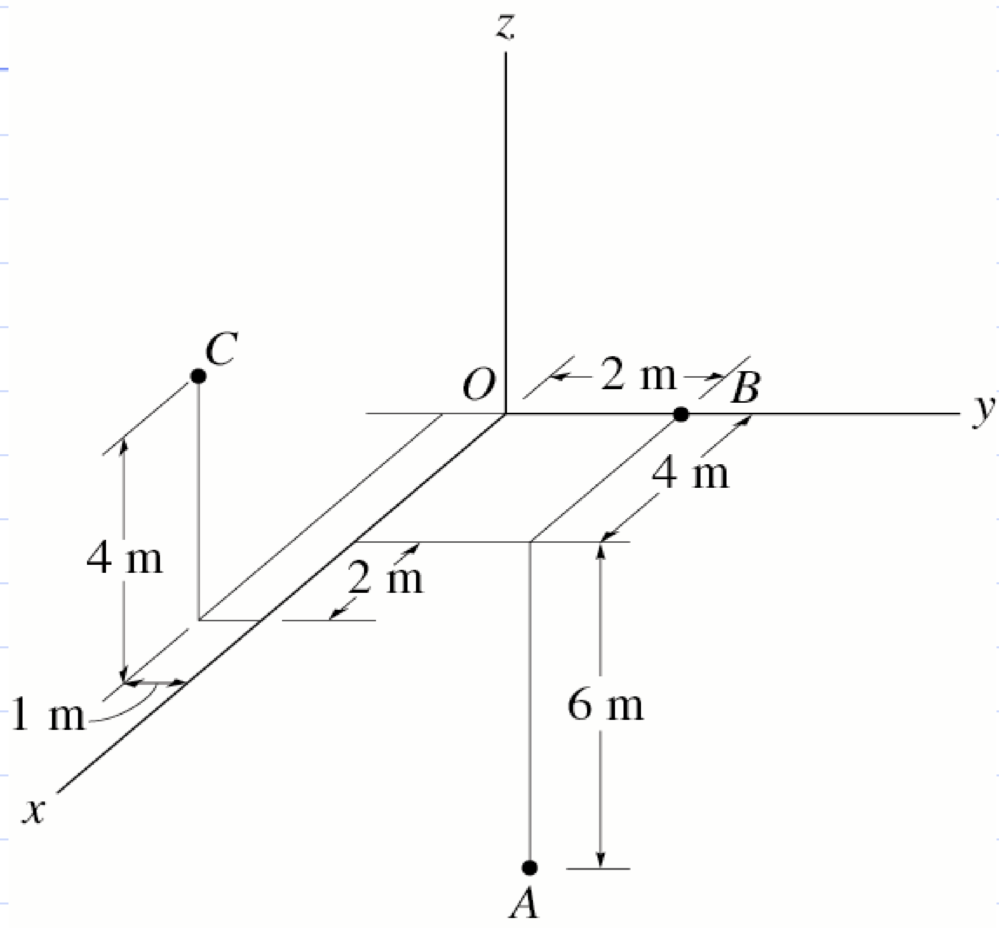
$$\cos\beta_2 = \frac{650}{700} \quad \Rightarrow \quad \beta_2 = \cos^{-1}\left(\frac{650}{700}\right) = 21.8^{\circ}$$

$$\cos\gamma_2 = \frac{150}{700} \quad \Rightarrow \quad \gamma_2 = \cos^{-1}\left(\frac{150}{700}\right) = 77.6^{\circ}$$



z
$\gamma_2 = 77.6°$
$F_2 = 700$ N
$\beta_2 = 21.8°$
$F_R = 800$ N
y
$\alpha_2 = 108°$
$F_1 = 300$ N
x

Position Vectors



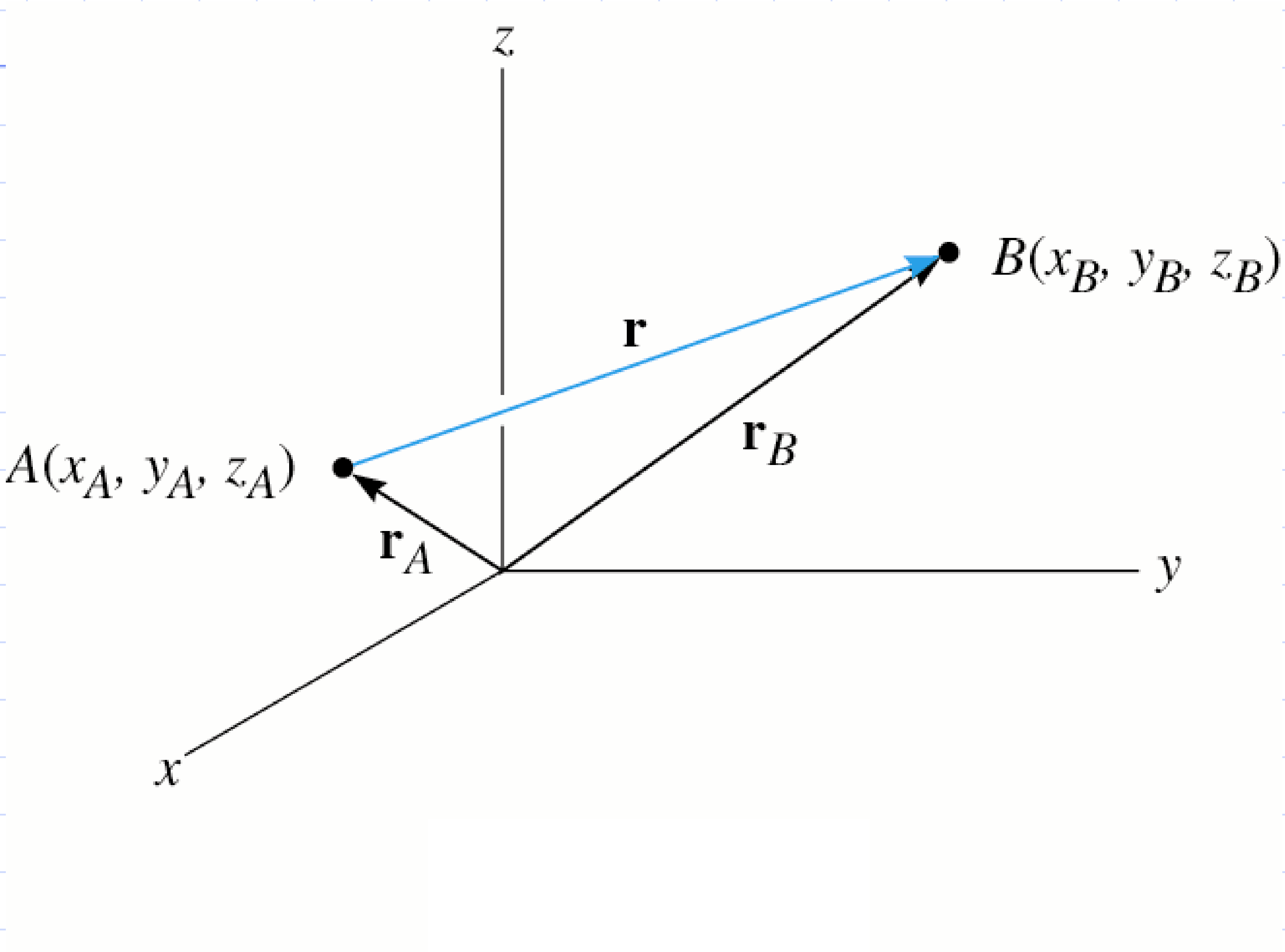
Coordinates

1. Right hand coordinate system
2. z - positive upwards
3. Position vector given by:

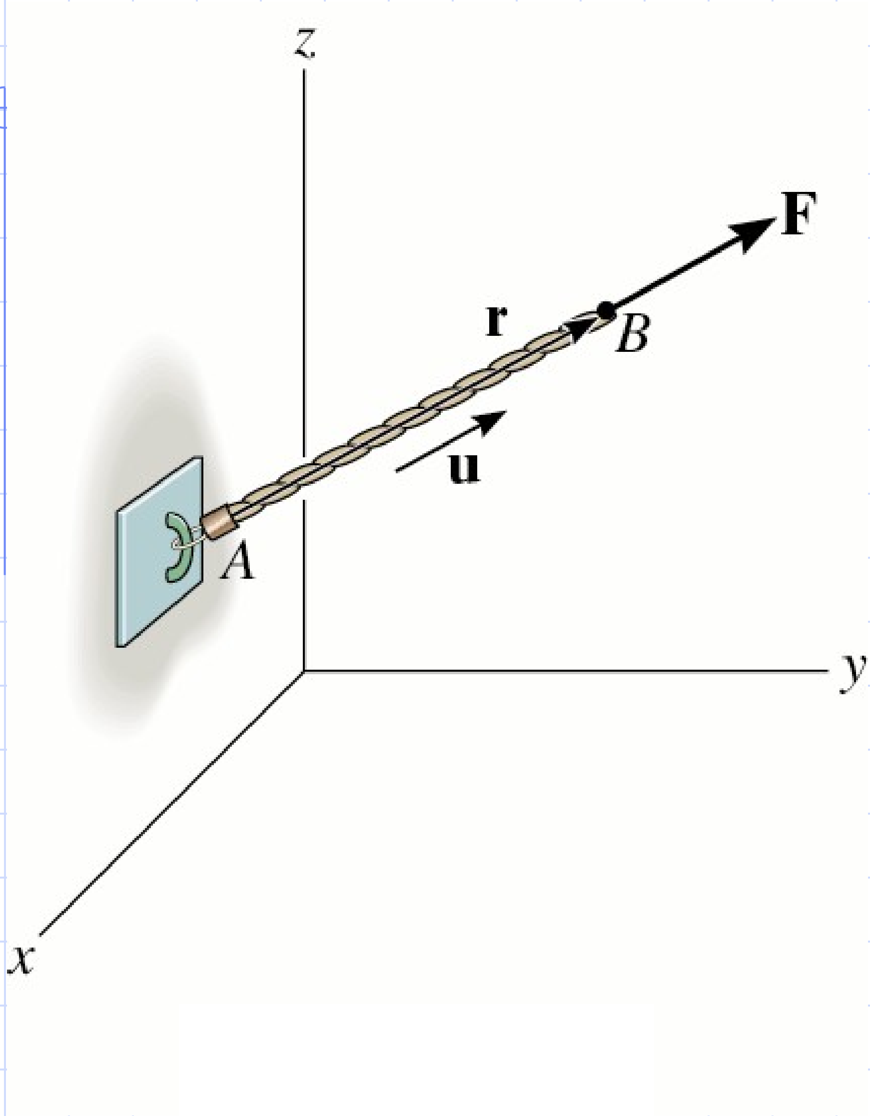
Cartesian Vector Form

$$\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$$

Relative Position Vectors

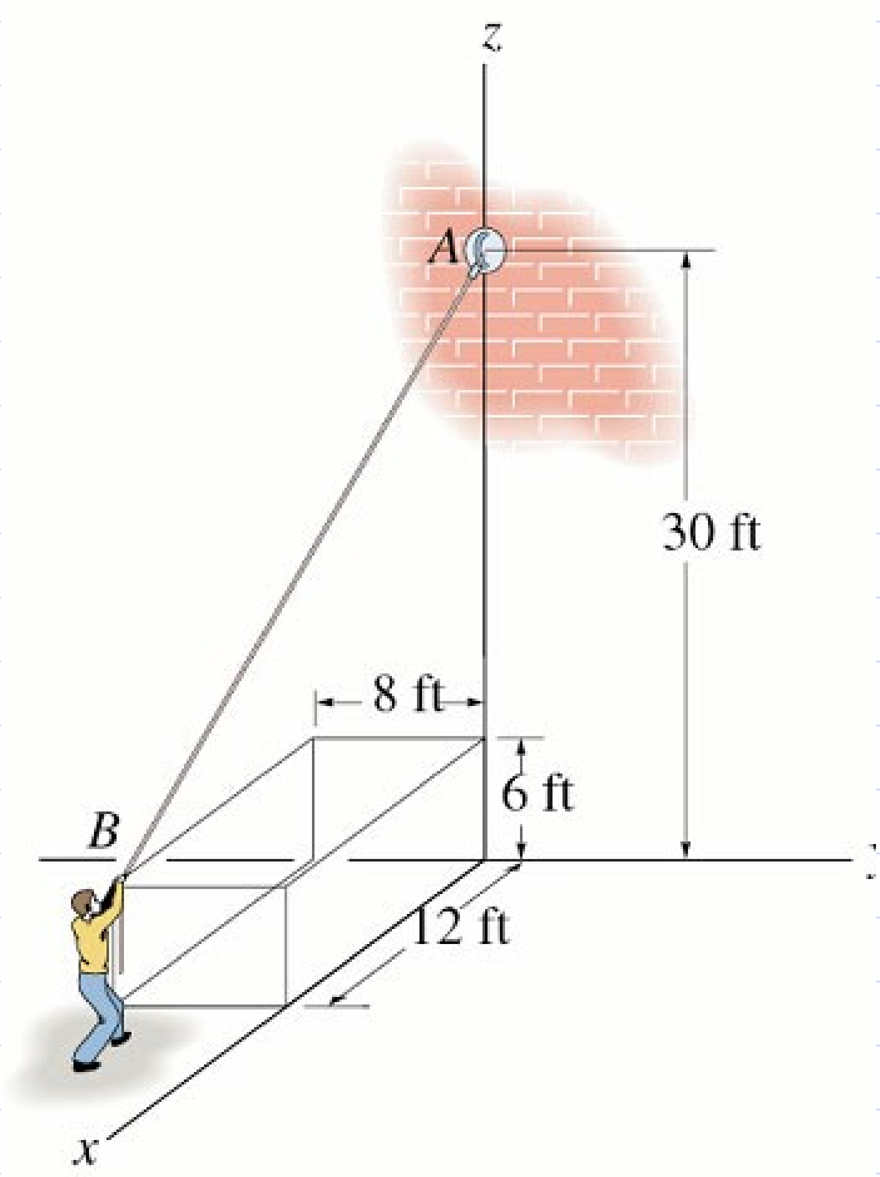


Force Along a Line



$$\vec{F} = F\hat{u} = F\left(\frac{\vec{r}}{r}\right)$$

Example



The man shown in the figure pulls on a cord with a force of 70 lb. Represent the force acting on support A as a Cartesian vector and determine its direction.

Position Vector

$$\vec{r}_{AB} = (x_B - x_A)\hat{i} + (y_B - y_A)\hat{j} + (z_B - z_A)\hat{k}$$

Locate Points of Interest:

$$A(0\text{ft}, 0\text{ft}, 30\text{ft}) \qquad B(12\text{ft}, -8\text{ft}, 6\text{ft})$$

$$\vec{r}_{AB} = (12 - 0)\hat{i} + (-8 - 0)\hat{j} + (6 - 30)\hat{k}$$

$$\vec{r}_{AB} = \{12\hat{i} - 8\hat{j} - 24\hat{k}\}\ \text{ft}$$

Unit Vector

$$\vec{r}_{AB} = \left\{ 12\hat{i} - 8\hat{j} - 24\hat{k} \right\} \text{ ft}$$

$$r_{AB} = \sqrt{(12)^2 + (-8)^2 + (-24)^2} = 28 \text{ ft}$$

$$\hat{u}_{AB} = \frac{\vec{r}_{BA}}{r_{BA}} = \frac{1}{28}\left(12\hat{i} - 8\hat{j} - 24\hat{k}\right) = \frac{3}{7}\hat{i} - \frac{2}{7}\hat{j} - \frac{6}{7}\hat{k}$$

Force Vector

$$\hat{u}_{AB} = \frac{3}{7}\hat{i} - \frac{2}{7}\hat{j} - \frac{6}{7}\hat{k}$$

$$\vec{F} = F\hat{u}_{AB} = 70\,\mathrm{lb}\left(\frac{3}{7}\hat{i} - \frac{2}{7}\hat{j} - \frac{6}{7}\hat{k}\right)$$

$$\vec{F} = \left(30\hat{i} - 20\hat{j} - 60\hat{k}\right)\,\mathrm{lb}$$

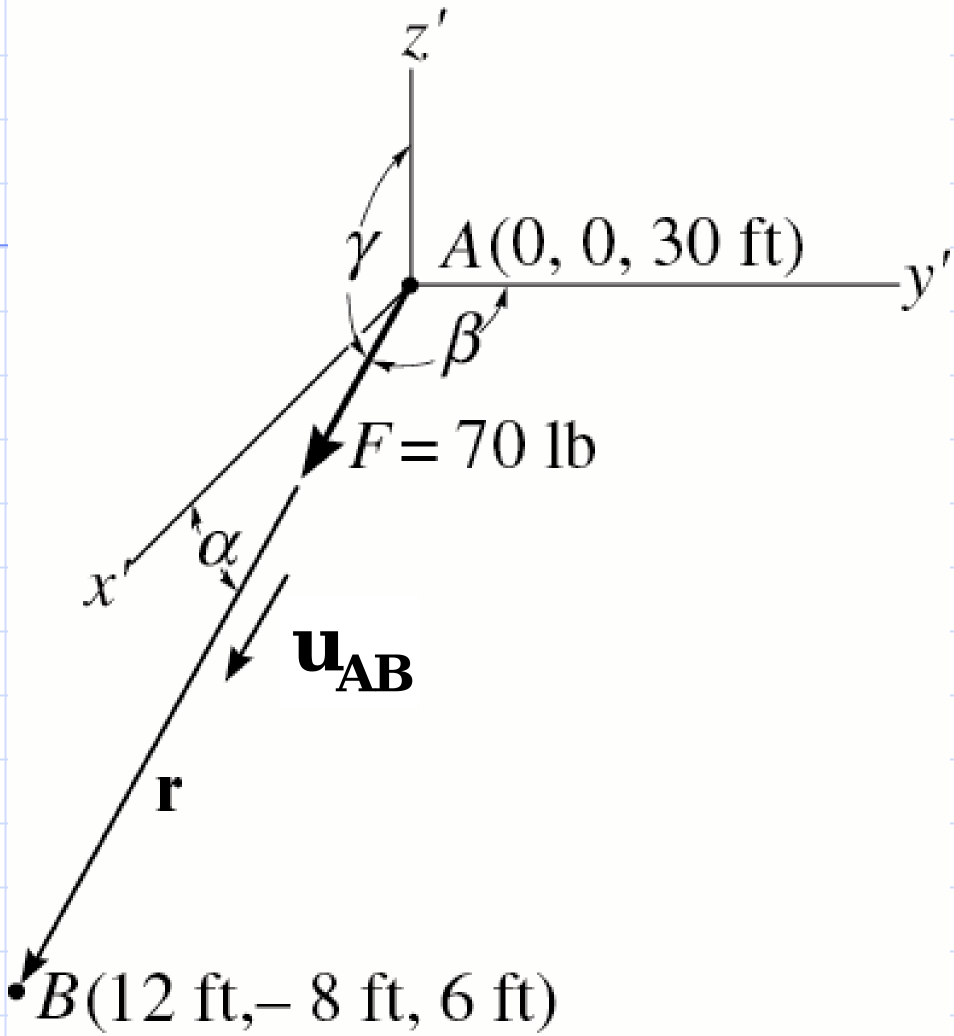
Direction Angles

$$\hat{u}_{AB} = \frac{3}{7}\hat{i} - \frac{2}{7}\hat{j} - \frac{6}{7}\hat{k}$$

$$\cos\alpha = \frac{3}{7} \quad \Rightarrow \quad \alpha = 64.6^{o}$$

$$\cos\beta = -\frac{2}{7} \quad \Rightarrow \quad \beta = 107^{o}$$

$$\cos\gamma = -\frac{6}{7} \quad \Rightarrow \quad \gamma = 149^{o}$$

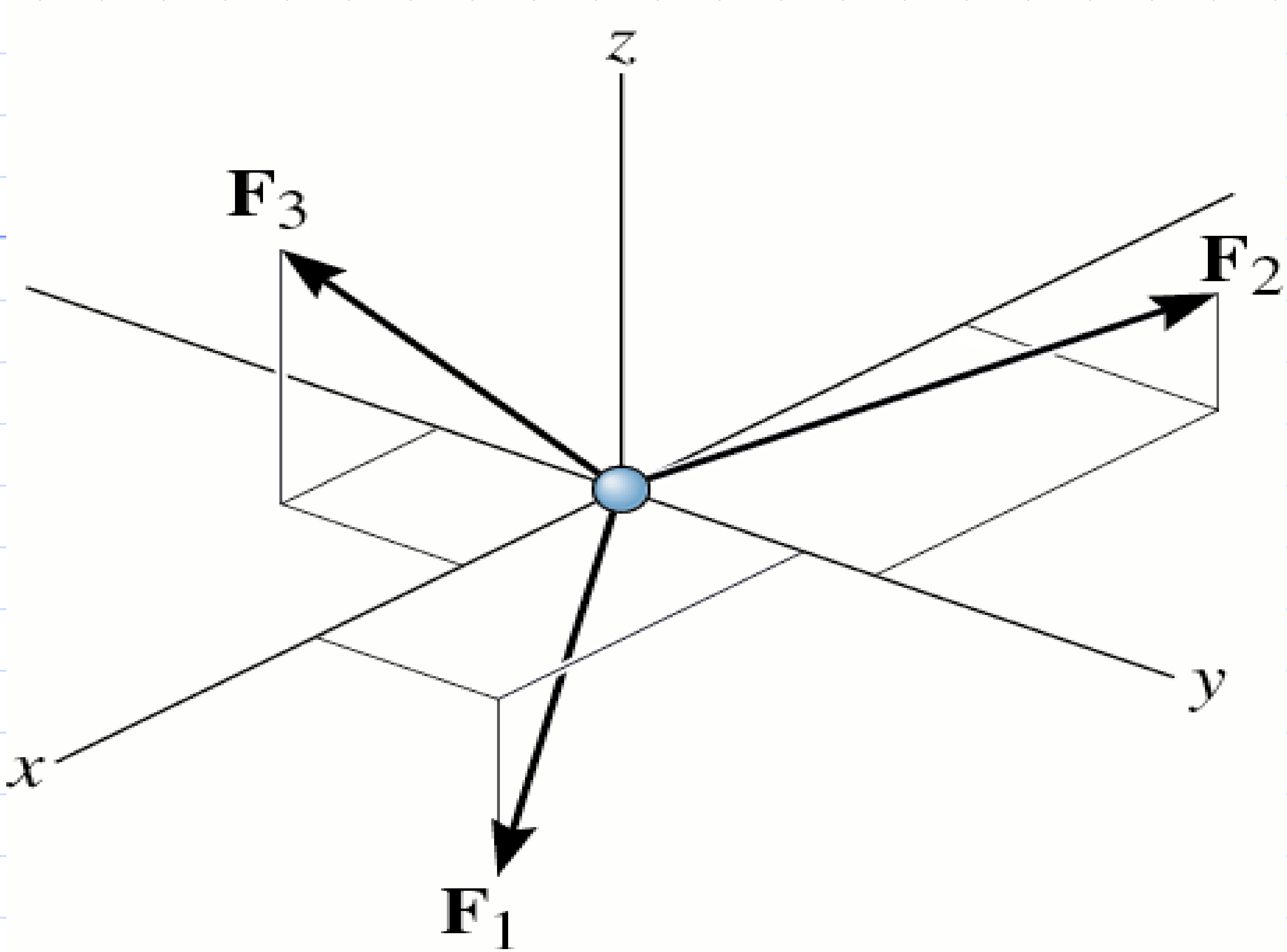


z'
γ
A(0, 0, 30 ft)
y'
β
F = 70 lb
α
x'
u_AB
r
B(12 ft,– 8 ft, 6 ft)

3D Equilibrium

$$\sum \vec{F} = 0$$

$$\textit{where } \sum \mathrm{F} \textit{ is the vector sum of all forces acting on the particle.}$$



z
$\mathbf{F}_3$
$\mathbf{F}_2$
y
x
$\mathbf{F}_1$

Three-Dimensional Force System

Use **i** , **j**, and **k** unit vectors.

$$\sum \vec{\mathrm{F}} = 0$$

$$\sum F_x \hat{\mathrm{i}} + \sum F_y \hat{\mathrm{j}} + \sum F_z \hat{\mathrm{k}} = 0$$

3D Equilibrium Equations

$$\sum F_x = 0$$

$$\sum F_y = 0$$

$$\sum F_z = 0$$

alar equations of equilibrium require that the gebraic sum of the x, y and z components of all rces acting on a particle be equal to zero.

3D Equilibrium Equations

$$\sum F_x = 0$$

$$\sum F_y = 0$$

$$\sum F_z = 0$$

ree equations means only three unknowns car
lved for from a single FBD.

Procedure for Analysis

Free-Body Diagram

1. Establish the x, y, and z axes in any suitable orientation.
2. Label all known and unknown force magnitudes and directions on the FBD.
3. The sense of an unknown force may be assumed.

Procedure for Analysis

Equations of Equilibrium

1. Resolve force vectors into Cartesian components.
2. Apply equations of equilibrium.

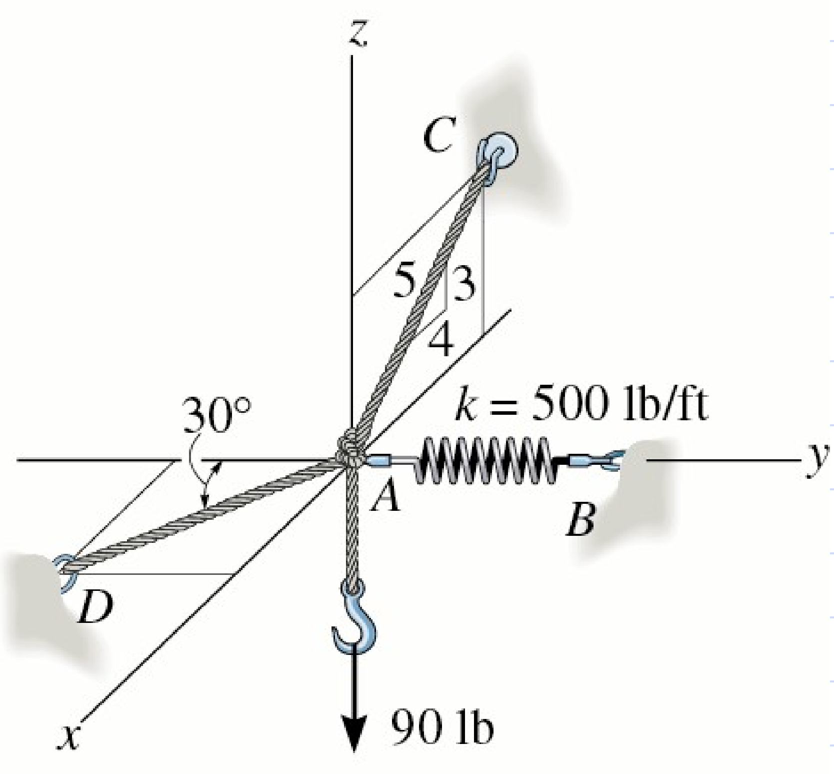
$$\sum F_x = 0, \quad \sum F_x = 0, \quad \text{and} \quad \sum F_y = 0$$

3. If solution yields a negative result the force is in the opposite sense of that shown on the FBD.


A
B
C
D

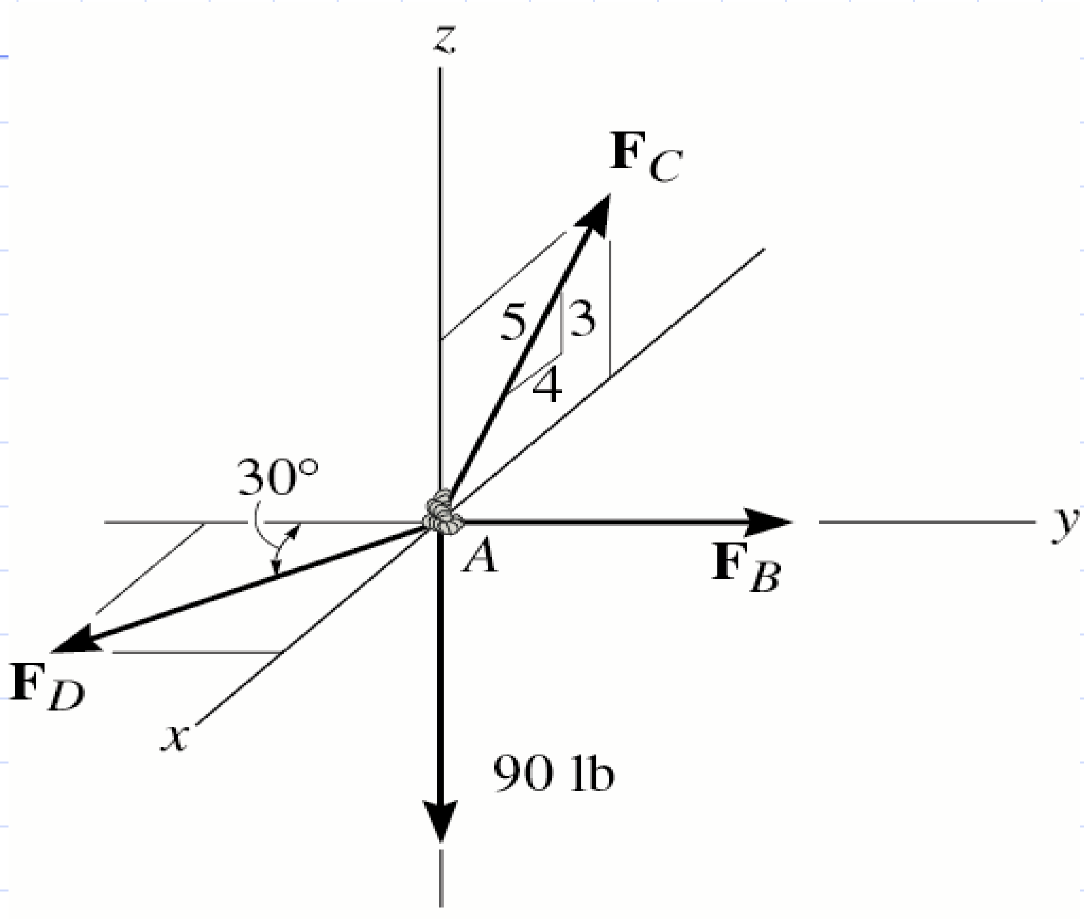

W
$\mathbf{F}_B$
$\mathbf{F}_D$
$\mathbf{F}_C$

Example



A 90 lb load is suspended from the hook as shown. The load is supported by two cables and a spring with k=500 lb/ft. Determine the force in the cables and the stretch of the spring for equilibrium. Cable AD lies in the x-y plane and cable AC lies in the x-z plane.

Free Body Diagram



Equilibrium Equations

$$\sum F_x = 0 \quad F_D \sin 30^o - \frac{4}{5} F_C = 0$$

$$\sum F_y = 0 \quad - F_D \cos 30^o + F_B = 0$$

$$\sum F_z = 0 \quad \frac{3}{5} F_C - 90 lb = 0$$

Solution

$$F_D \sin 30^o - \frac{4}{5} F_C = 0$$

$$-F_D \cos 30^o + F_B = 0$$

$$\frac{3}{5} F_C - 90lb = 0$$

$$\boxed{\begin{aligned} F_C &= 150lb \\ F_D &= 240lb \\ F_B &= 208lb \end{aligned}}$$

Stretch

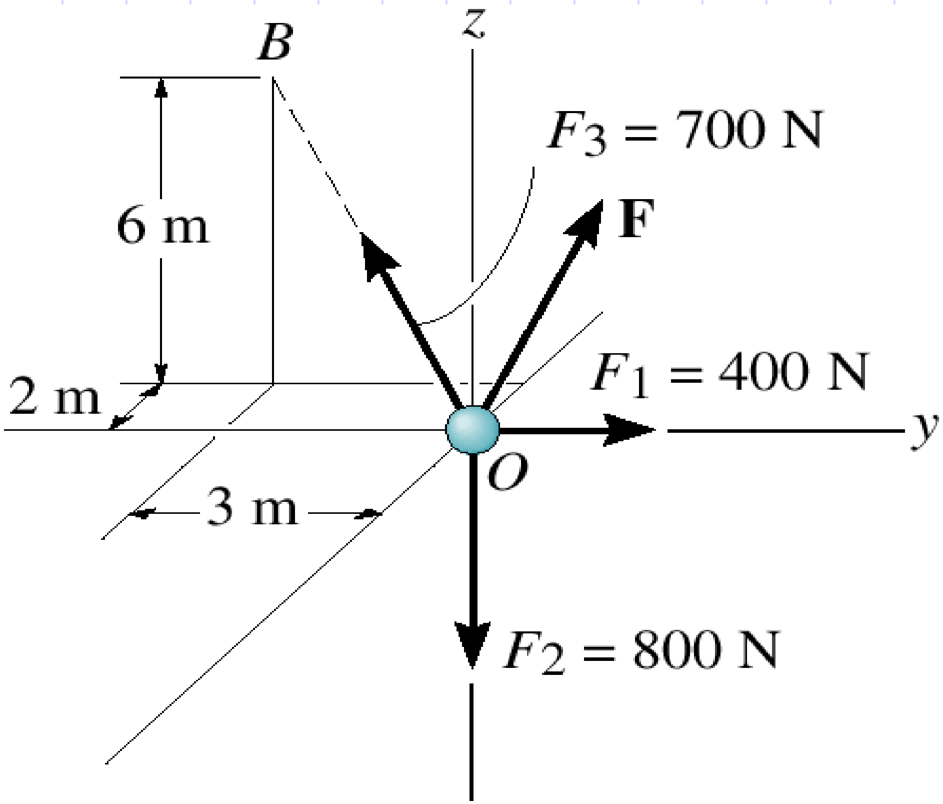
$$F_B = 208\,\mathrm{lb}$$

$$F_B = k s_{AB}$$

$$208\,\mathrm{lb} = 500\frac{\mathrm{lb}}{\mathrm{ft}}\, s_{AB}$$

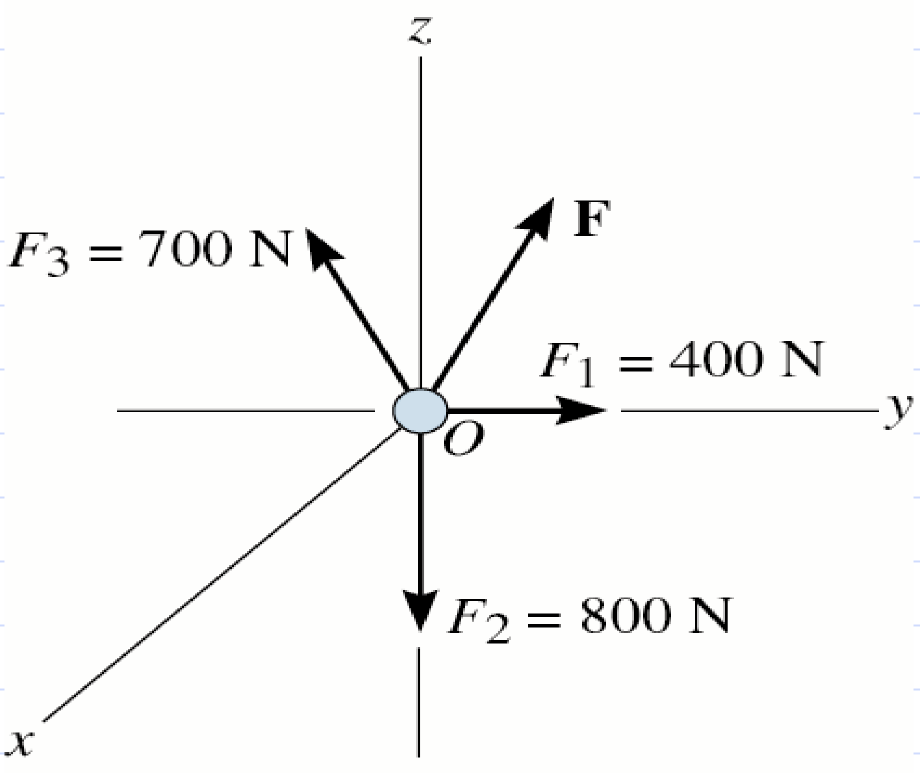
$$s_{AB} = 0.416\,\mathrm{ft}$$

Example



Determine the magnitude and coordinate direction angles of the force, **F**, required for equilibrium of particle O.

Free Body Diagram



Vector Forces

$$\vec{F}_1 = \left(400\hat{j}\right) \text{N}$$

$$\vec{F}_2 = \left(-800\hat{k}\right) \text{N}$$

$$\vec{F}_3 = F_3\left(\frac{\vec{r}_{OB}}{r_{OB}}\right) = 700\text{N}\left[\frac{-2\hat{i} - 3\hat{j} + 6\hat{k}}{\sqrt{(-2)^2 + (-3)^2 + (6)^2}}\right]$$

$$\vec{F}_3 = \left(-200\hat{i} - 300\hat{j} + 600\hat{k}\right) \text{N}$$

$$\vec{F} = F_x\hat{i} + F_y\hat{j} + F_z\hat{k}$$

Equilibrium

$$\sum \vec{F} = 0$$

$$\vec{F}_1 + \vec{F}_2 + \vec{F}_3 + \vec{F} = 0$$

$$400\hat{j} - 800\hat{k} - 200\hat{i} - 300\hat{j} + 600\hat{k}$$
$$+ F_x\hat{i} + F_y\hat{j} + F_z\hat{k} = 0$$

$$\sum F_x = 0 \qquad -200 + F_x = 0$$

$$\sum F_y = 0 \qquad 400 - 300 + F_y = 0$$

$$\sum F_z = 0 \qquad -800 + +600 + F_z = 0$$

Solution

$$-200 + F_x = 0 \Rightarrow F_x = +200\text{ N}$$

$$400 - 300 + F_y = 0 \Rightarrow F_y = -100\text{ N}$$

$$-800 + +600 + F_z = 0 \Rightarrow F_z = +200\text{ N}$$

Solution

$$\mathbf{F} = \left(200\hat{\mathbf{i}} - 100\hat{\mathbf{j}} + 200\hat{\mathbf{k}}\right)\ \mathrm{N}$$

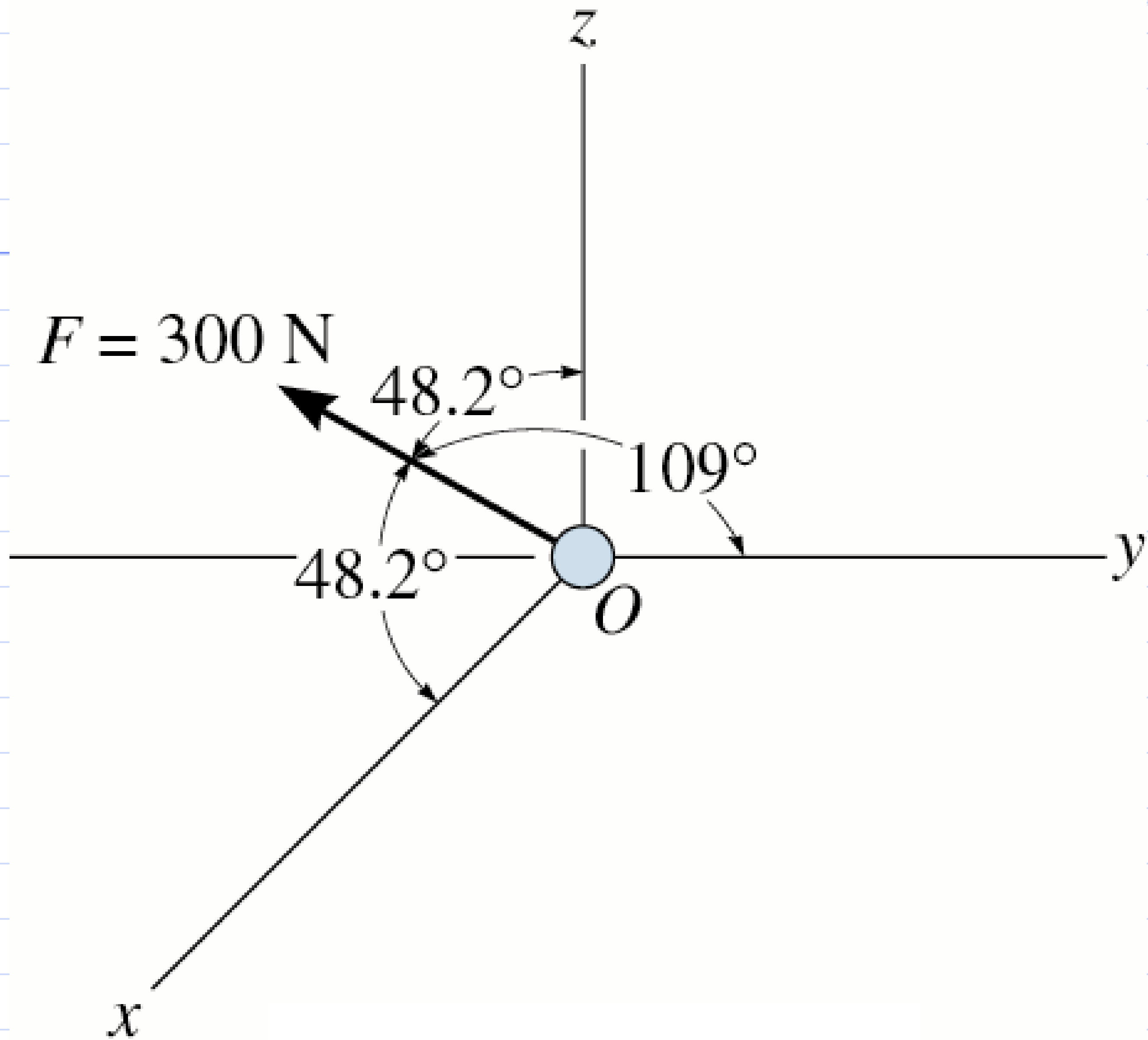
$$F = \sqrt{(200)^2 + (-100)^2 + (200)^2} = 300\ \mathrm{N}$$

$$\hat{\mathbf{u}}_F = \frac{2}{3}\hat{\mathbf{i}} - \frac{1}{3}\hat{\mathbf{j}} + \frac{2}{3}\hat{\mathbf{k}}$$

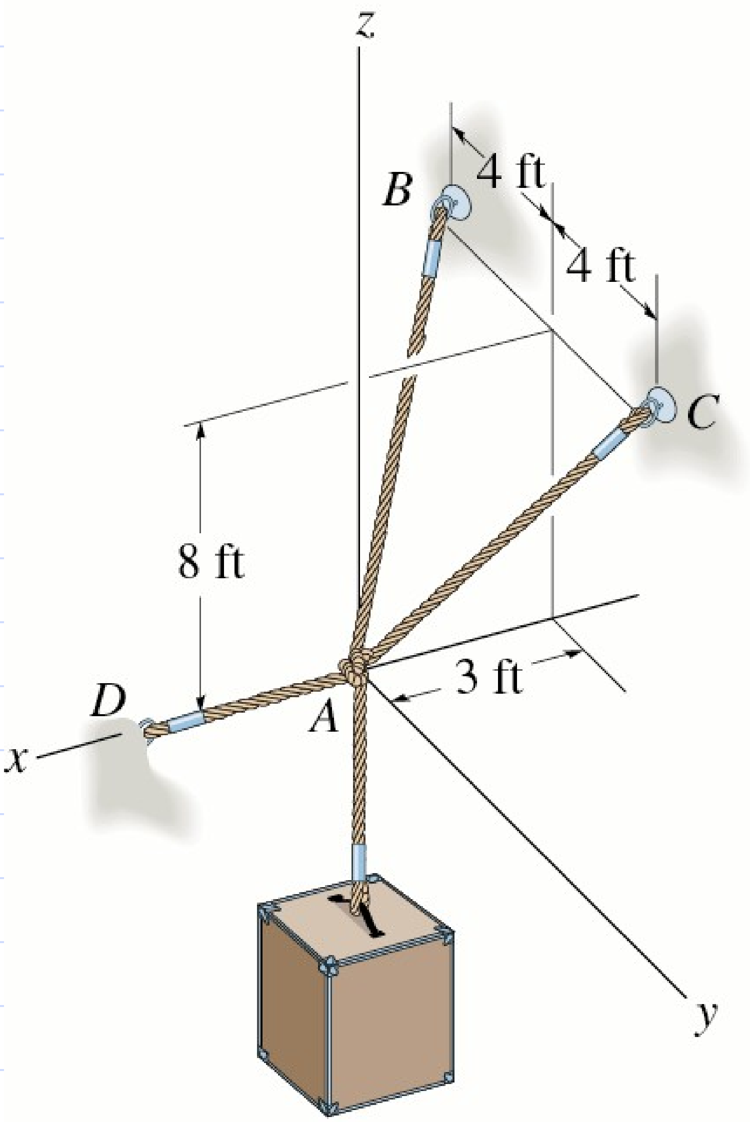
$$\alpha = \cos^{-1}\left(\frac{2}{3}\right) = 48.2^{o}$$

$$\beta = \cos^{-1}\left(-\frac{1}{3}\right) = 109^{o}$$

$$\gamma = \cos^{-1}\left(\frac{2}{3}\right) = 48.2^{o}$$



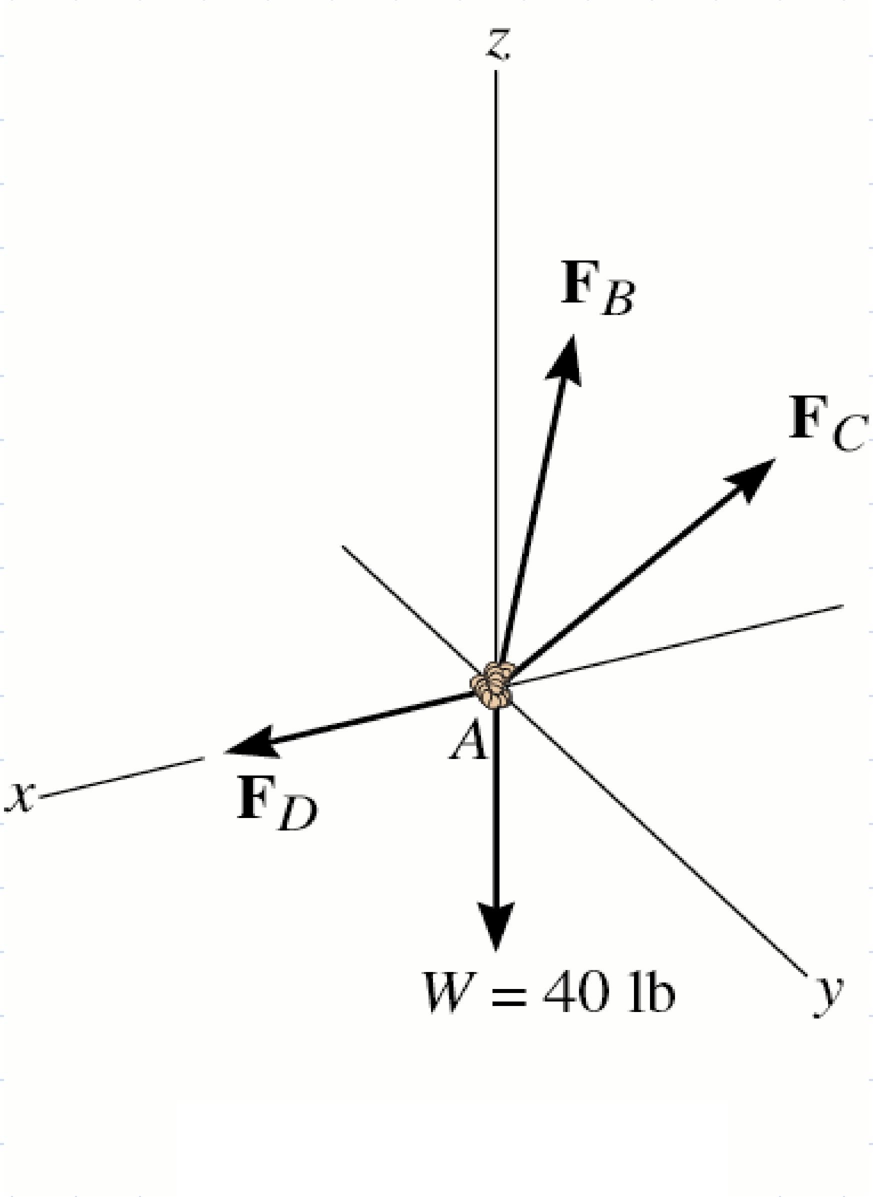
z
F = 300 N
48.2°
109°
48.2°
y
O
x



Example

Determine the force in each cable used to support the 40 lb crate.

Free Body Diagram



Express each force in Cartesian vector form.

The locations (in feet) of the three points are:

A (0, 0, 0) B (-3, -4, 8)

C (-3, 4, 8)

$$\vec{F}_B = F_B \hat{u}_{AB}$$

$$\vec{F}_C = F_C \hat{u}_{AC}$$

$$\vec{F}_D = F_D \hat{u}_{AD}$$

Vector Forces

$$\vec{F}_B = F_B\left(\frac{\vec{r}_{AB}}{r_{AB}}\right) = F_B\left[\frac{-3\hat{i} - 4\hat{j} + 8\hat{k}}{\sqrt{(-3)^2 + (-3)^2 + (8)^2}}\right]$$

$$\vec{F}_B = -0.318 F_B\,\hat{i} - 0.424 F_B\,\hat{j} + 0.848 F_B\,\hat{k}$$

$$\vec{F}_C = F_C\left(\frac{\vec{r}_{AC}}{r_{AC}}\right) = F_C\left[\frac{-3\hat{i} + 4\hat{j} + 8\hat{k}}{\sqrt{(-2)^2 + (-3)^2 + (8)^2}}\right]$$

$$\vec{F}_C = -0.318 F_C\,\hat{i} + 0.424 F_C\,\hat{j} + 0.848 F_C\,\hat{k}$$

$$\vec{F}_D = F_D\hat{i}$$

$$\vec{W} = \left(-40\hat{k}\right)\text{lb}$$

Equilibrium

$$\sum \vec{F} = 0$$

$$\vec{F}_B + \vec{F}_C + \vec{F}_D + \vec{W} = 0$$

$$-0.318F_B\hat{i} - 0.424F_B\hat{j} + 0.848F_B\hat{k}$$
$$-0.318F_C\hat{i} + 0.424F_C\hat{j} + 0.848F_C\hat{k} + F_D\hat{i} - 40\hat{k} = 0$$

$$\sum F_x = 0 \qquad -0.318F_B - 0.318F_C + F_D = 0$$
$$\sum F_y = 0 \qquad -0.424F_B + 0.424F_C = 0$$
$$\sum F_z = 0 \qquad 0.848F_B + 0.848F_C - 40 = 0$$

Solution

$$-0.318F_B - 0.318F_C + F_D = 0$$

$$-0.424F_B + 0.424F_C = 0$$

$$0.848F_B + 0.848F_C - 40 = 0$$

$$F_B = F_C = 23.6\,\mathrm{lb}$$

$$F_D = 15.0\,\mathrm{lb}$$